Universally optimal distribution of points on spheres

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Abstract

How should one distribute a certain number of points over a sphere, so that they are well separated? One natural method is energy minimization: put an electric charge on each point and let them repel each other. Being mathematicians, we can choose any (reasonable) force law instead of the electric force. As the potential function gets steeper, this problem turns into sphere packing, but all force laws are of interest. The optimal configuration generally depends on the force law chosen, but in some rare cases it does not; we call those universally optimal configurations. In $\mathbb{R}^3$ there are three universal optima, namely the vertices of a regular tetrahedron, octahedron, or icosahedron. In higher dimensions, there are even more fascinating examples (for example, the E_8 root system, the Leech lattice minimal vectors, and a configuration of 27 points in $\mathbb{R}^6$ related to the 27 lines on a cubic surface). In this talk I'll explain what's known and what's not about universal optimality. This is joint work with Abhinav Kumar.

This talk should be accessible to graduate students.