Gaps for regular maps

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007 Kemeny Hall, 4:00 pm

(Tea 3:30 pm 300 Kemeny Hall)

Abstract

A regular map of type $\{k, m\}$ is a map with rotational symmetry of order k about vertices, m about centers of faces, and 2 about midpoints of edges. The only examples in the sphere are the five platonic solids. There is a natural correspondence between $\{k, m\}$ regular maps and groups generated by two elements of orders k and m whose product has order 2, so studying regular maps is equivalent to studying such groups. This correspondence can be exploited to determine all regular maps of small genus, the most recent effort last year by Marston Conder on MAGMA reaching genus 100.

Such lists have shown that, although there are trivial ways to construct regular maps on orientable surfaces of all genera, there are also gaps for various types of maps: nonorientable, chiral (not admitting an orientation-reversing automorphism), and nondegenerate (no multiple edges in the primal or dual graphs). An infinite set of nonorientable gaps was explained three years ago in a ground-breaking paper by Breda, Nedela, and Siran. In this talk, we will describe our work with Conder and Siran determining all orientable maps of genus $p + 1$ where p is prime. This is the first classification theorem for an infinite set of orientable genera, since the study of regular maps began in the 1920s under Brahana, followed by Coxeter and others. The classification reveals deep connections between nonorientability, chirality, and degeneracy. It uses a combination of new ideas from elementary group theory together with classical results such as Schur's Theorem, Ito's Theorem, and the the Suzuki-Wong classification of nonsolvable, almost Sylow-cyclic groups.

This talk should be accessible to graduate students.

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