Geometric Quantization and Operator K-Theory

Jeff Fox
University of Colorado

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102 Bradley Hall, 4:00 pm
(Tea 3:30 pm Math Lounge)

Abstract

One of the most important ideas in the unitary representation theory of Lie groups is that of geometric quantization. It was first proposed by Kostant and then developed by Kirillov to give a complete description of the unitary dual for nilpotent Lie groups. The scope of geometric quantization was expanded by a number of different people in a number of different directions and it is fair to say that geometric quantization has been a guiding principle in almost all of the work done in the last 30 years on the representation theory of locally compact groups.

In this talk we describe the picture that geometric quantization gives for the simple Lie group $SU(1,1)$. Semisimple Lie groups have no finite dimensional irreducible unitary representations, but do have finite dimensional non-unitary representations (the"Unitary Trick"of Weyl). These finite dimensional non-unitary representations control the infinite dimensional unitary representation theory of semisimple Lie groups in a very concrete way via the mechanisms of quantization.

If G is a locally compact group, Kasparov introduced the ring of Fredholm representations, $R(G)$. This ring, or more precisely a certain idempotent known (sometimes) affectionately as Kasparov's gamma element,$\gamma \in R(G)$, has been a decisive tool in computing the K-theory of the reduced C^*-algebra $C_r^*(G)$. We will describe a conjectural correspondence between geometric quantization and Kasparov's $R(G)$ for semisimple Lie groups. One advantage of this correspondence, besides giving a fairly natural geometric construction of elements in $R(G)$, is that- in some cases- it is also true.

This talk should be accessible to graduate students.