Duals of group multiplications and group actions

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October 01, 1998
102 Bradley Hall, 4:00 pm
(Tea 3:30 pm Math Lounge)

Abstract

A multiplication on an object X is a map $m : X \times X \to X$ with certain properties which capture the axioms of associativity, unit and inverses of a group. An action of a group G on an object X is given in terms of a map $a : G \times X \to X$. These notions are interesting in many different contexts, e.g., when the objects are topological spaces, groups, monoids, etc. The purpose of this talk is to answer the question: When can these notions be dualized, i.e., when can all the arrows be reversed in these definitions, and what results are obtained in the different settings? There is a strong motivation from algebraic topology where the duals in the case of topological spaces are well known constructions such as suspensions. To define a "comultiplication" we need the notion of a coproduct which is dual to that of a product. This notion is available for topological spaces, groups (the free product "$*$") and monoids. Thus a comultiplication on a group G is a homomorphism $m : G \to G * G$ whose composition with the projections $G * G \to G$ is the identity. A coaction of G on a group K is similarly defined by a homomorphism $G \to G * K$. In this talk we will describe the structure of groups (and monoids) which admit a comultiplication or a coaction.

Although the motivation comes from topology, the talk will be purely algebraic. Only a little group theory will be assumed.

This talk should be accessible to graduate students.

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