The Ropelength of Knots and Links

Jason Cantarella
University of Massachusetts

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102 Bradley Hall, 4:00 pm
(Tea 3:30 pm Math Lounge)

Abstract

How much rope does it take to tie a knot? And what do the ropelength minimizing configurations of a knot look like?

The answer to these questions is of interest in molecular biology, where the speed of knotted strands of DNA moving through a gel is linked to the minimum ropelength of their knot types.

In this talk, we'll present a survey of some recent results on ropelength-minimizing knots and links, including joint work with Rob Kusner, John Sullivan, Heather Johnston, and Joe Fu.

We'll prove that ropelength minimizers exist and are $C^{1,1}$ in every knot and link type. We'll discuss "large-scale" bounds on ropelength in terms of crossing number, showing that for every knot,

$$.4C(K)^{3/4} < R(K) < 25C(K)^2,$$

and constructing knots and links which show that the exponent of the lower bound is sharp. And we'll discuss a new family of ropelength bounds which allow us to construct a family of tight links, and to improve the best known lower bounds on the ropelength of a nontrivial knot.

This talk should be accessible to graduate students.

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