The coset poset and probabilistic zeta function of a finite group

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(Tea 3:30 pm Math Lounge)

Abstract

For a finite group G and a non-negative integer s, let P(G,s) be the probability that a randomly chosen s-tuple generates G. Philip Hall gave an explicit formula for P(G,s), exhibiting the latter as a finite Dirichlet series. One can therefore speak of P(G,s) for an arbitrary complex number s. The reciprocal of this function of s is called the zeta function of G.

The present work arose from an attempt to understand the value of the zeta function at s = -1. More precisely, I wanted to explain some surprising divisibility properties of P(G,-1), which is an integer, that I observed empirically. The explanation turns out to involve topology. Specifically, one is led to study the topological properties of the coset poset of G, consisting of proper cosets xH ordered by inclusion.

This talk should be accessible to graduate students.