Numerical cubature from geometry and coding theory

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L02 Carson Hall, 4:00 pm
(Tea 3:30 pm Math Lounge)

Abstract

The numerical cubature problem is the generalization to higher dimensions of integration methods such as Simpson's rule. Given a measure μ on R^n, a t-cubature formula is a finite set C such that the integral of any polynomial P of degree t with respect to μ equals a weighted sum over values on C. The main interest is in cubature formulas with few points, with positive weights, and without points outside of the domain of μ. Gaussian quadrature satisfies all three conditions in one dimension, but the problem is already open-ended in two dimensions and increasingly non-trivial in higher dimensions.

I will discuss new methods for the cubature problem coming from error-correcting codes, symplectic moment maps, and lattice packings of discretized convex bodies. The methods yield many new explicit, efficient, positive, interior, cubature formulas for the most standard choices of μ. In one context, they also lead to an interesting local lower bound on the number of points needed for cubature.

This talk should be accessible to graduate students.

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