Complexity and Good Spaces

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Thursday, April 28, 2005
L02 Carson Hall, 4:00 pm
(Tea 3:30 pm Math Lounge)

Abstract

In algebraic topology, we like to build new spaces out of ones we already understand. One procedure for doing this is called the homotopy colimit. For any space A, we can form the smallest collection of spaces which contains A and which is closed under the operation of taking homotopy colimits. Such a collection is called the *closed class* generated by A, and it is denoted $\mathcal{C}(A)$.

Now if $X \in \mathcal{C}(A)$, then X can be obtained from A by repeating the homotopy colimit operation, perhaps infinitely many times. Indeed, X could be obtained in many different ways. The A-*complexity* of X with respect to A is the minimum number of times you need to repeat the homotopy colimit operation before getting X. It is denoted $\kappa_A(X)$.

It may happen that $\mathcal{C}(A)$ is, in addition and by accident, closed under another construction—extensions by fibrations. If this is the case, then we call A a *good space*. Examples suggest that complexity with respect to good spaces is relatively small. In this talk, I'll characterize good space and use the characterization to prove that this intuition is correct —complexity with respect to good spaces is (more or less) always finite or (at worst) countably infinite.

This is joint work with Michele Intermont. I will try to keep the really technical stuff under the rug, so that the talk will be accessible to graduate students who are comfortable with CW complexes.

This talk should be accessible to graduate students.