Index theory and noncommutative geometry

Alan Paterson
University of Mississippi

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(Tea 3:30 pm Math Lounge)

Abstract

The Atiyah–Singer index theorem associates with an elliptic differential operator two numbers, one defined by analytic means (the analytic index) and the other defined by topological means (the topological index), and shows (e.g. via K-theory) that the two numbers are the same. The talk will begin by illustrating the index theorem with the Gauss–Bonnet theorem, which corresponds to the case of the de Rham operator. The original index theorem works within a commutative framework (e.g. $C(X)$) but Alain Connes showed that many naturally ocurring situations require developing index theory in the non-commutative context (using C^*-algebras and groupoids). In particular, he proved an index theorem for measured foliations. A good example of such a foliation is the Kronecker foliation of the torus; this involves an irrational rotation C^*-algebra and the leaf equivalence relation groupoid. We conclude the talk by discussing briefly a general index theorem formulated within the framework of groupoid actions and the KK-theory of C^*-algebras.

This talk should be accessible to graduate students.