An "algebraic K-theory" of finite groups: The unstable case and reclusive primes

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Abstract

Persi Diaconis asked me if it might be possible to develop a "K-theory of finite groups" based on n-tuples of generating sets, which would replace rows of invertible matrices (the bases of free modules). I found a way to construct groups that have a universal mapping property analogous to those of free modules: A group H is "n-homogeneous" if for every pair of ordered generating n-tuples, there exists a unique automorphism of the group which takes the first to the second.

It turns out that such groups were discovered by B. H. Neumann & H. Neumann in 1951. Given a finite group G generated by r elements, there is a natural way to construct an n-homogeneous group $H(n, G)$ for $n > r$. The automorphism groups of these groups play the role of the group of invertible matrices in K-theory, and one develops the stable algebra of these groups as n goes to infinity, as in traditional algebraic K-theory.

Recently I have been looking at the unstable case, rather than the stable case arising in K-theory. Really interesting things happen when r is the minimum number of generators of the group G.

The general construction gives a group $H(r, G)$ that has r generators all of the same order. For $n > r$, it is trivial to see that this number is the exponent of G. The question then arises: what is this number for $n = r$? After trying unsuccessfully to prove that it was the exponent, I sought counterexamples, and to my surprise, I found them—surprisingly, so far all have turned out to be Frobenius groups. These considerations lead to an invariant that generalizes the classical Scharlau invariant. There are many interesting questions remaining even in this simplest case.

This talk should be accessible to graduate students.