Using Algebra and Algebraic Geometry to Study Curves

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007 Kemeny Hall, 4:00 pm
(Tea 3:30 pm 300 Kemeny Hall)

Abstract

Finding solutions to polynomial equations is one of the oldest questions in mathematics. Arithmetic geometry is the study of number theoretic questions using ideas from modern algebra and algebraic geometry. In particular we study solutions to systems of polynomial equations over rings such as the integers or number fields (finite extensions of $\mathbf{Q}$). This approach has had profound success in such varied ways as proving long open conjectures like Fermat's Last Theorem as well as finding applications in cryptography and error correction.

Perhaps the best known examples of arithmetic geometry come from the study of elliptic curves. The points on these curves have a beautiful, natural group structure which allows us to use results from algebra and algebraic geometry to study these groups, and in turn, the points on these curves. We will talk about these ideas as well as how to deal with more general curves which do not have this natural group structure.

This leads to the study of the Jacobian variety of a curve. This variety has a group structure on its points which is naturally associated to the points on the curve we started with. As such, the Jacobian variety fills the roll of the group structure on the elliptic curve for our study of general curves.

This talk should be accessible to graduate students.

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