What is so special about spheres?

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007 Kemeny Hall, 4:00 pm
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Abstract

The homotopy theory of ordinary spaces is powered by two basic theorems: the Hurewicz theorem and the Blakers-Massey theorem, both of which can be expressed as estimating the connectivity of certain spaces (the connectivity of a spaces is the largest dimension below the first nonzero homotopy group).

It can be useful, instructive and fun to axiomatize the object you are studying. This practice has a long history in homotopy theory, going back at least as far as the Eilenberg-Steenrod axioms for homology. In the 1960s, Quillen invented model categories to axiomatize homotopy theory. This framework does not have any notion of connectivity built into it, so that it is impossible to state, let alone prove, analogs of Hurewicz or Blakers-Massey in that very general context.

In this talk I'll speculate about abstracting the notion of connectivity in such a way that the Hurewicz and Blakers-Massey theorems will hold, and so perhaps make some progress toward axiomatizing our nice familiar homotopy theory. Along the way we'll see how a Galois correspondence involving topological spaces leads to the concept of closed classes, and we'll catch a glimpse of the Sullivan Conjecture.

This talk should be accessible to graduate students.