The automorphism tower of a group

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May 20, 1999
102 Bradley Hall, 4:00 pm
(Tea 3:30 pm Math Lounge)

Abstract

The automorphism tower of a group is obtained by computing its automorphism group, the automorphism group of *that* group and so on, iterating transfinitely. Each group maps into the next using inner automorphisms and one takes a direct limit at limit stages. The question is whether the process ever terminates in a fixed point, a group which is isomorphic to its automorphism group by the natural map. The classical result (Wielandt 1939) is that the automorphism tower of any finite centerless group terminates in finitely many steps. Hulse (1970) showed that the tower of any centerless polycyclic group terminates in countably many steps. Simon Thomas (1985), in an elegant argument, proved that the tower of any centerless group eventually terminates. Here, I will prove that every group has a terminating automorphism tower. After this, in a somewhat more metaphysical endeavor, I will discuss the essentially set-theoretic nature of the automorphism tower of a group, showing that it is consistent that a single group has, in various models of set theory, wildly diverging automorphism towers, whose heights can be almost arbitrarily specified.

This talk should be accessible to graduate students.

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