The First Eigenvalue of a Riemann Surface

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(Tea 3:30 pm Math Lounge)

Abstract

(joint work with Eran Makover)

In the '70's, McKean announced the following result, which he called "The Riemann Hypothesis for Riemann Surfaces":

Theorem: Let S be a Riemann surface of genus $g > 1$. Then the first eigenvalue of the Laplacian of S is at least $1/4$.

Unfortunately, McKean's result is quite false, as simple examples show. Nonetheless, one can still hope that the first eigenvalue of a "typical" Riemann surface is large.

We show that there is a positive constant C such that, for a randomly picked Riemann surface, the probability that the first eigenvalue is at least C tends to 1 as the genus goes to infinity. Part of the problem is to define what one means by a randomly picked Riemann surface, in a way that gives one control over the geometry. We do this by modeling picking a random Riemann surface on the problem of picking a random 3-regular graph.

This talk should be accessible to graduate students. 951243090