Free Torus Actions and Rational Homotopy

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(Tea 3:30 pm Math Lounge)

Abstract

Abstract: Circle actions, more generally torus actions, are of fundamental interest in the subject of transformation groups. Many ideas familiar from algebraic topology can be used to study these actions.

Example: The circle S^1 acts on the 3-sphere S^3. To see this, view the circle as unit-length complex numbers and the 3-sphere as the unit sphere in C^2. Then the action is by complex scalar multiplication. The orbit space of this action is S^2, and the projection of S^3 onto the orbit space is the well-known Hopf fibration $S^1 \to S^3 \to S^2$. The non-triviality of this fibration corresponds to the non-triviality of the action, in some sense.

In this talk, we study free torus actions mostly using cohomology and some tools from rational homotopy. We focus on the basic question of whether a space admits a free torus action, and if so what is the maximum size of torus that can act freely? We discuss some results related to the so-called toral rank conjecture of S. Halperin. We also mention briefly some work in progress. The talk will be a survey and should be accessible to graduate students who have taken Math 114.

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