Parameterized gauge theory and its applications

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102 Bradley Hall, 4:00 pm
(Tea 3:30 pm Math Lounge)

Abstract

For the last 20 years, mathematicians and physicists have been studying invariants of manifolds derived from the study of various non-linear partial differential equations, notably the Yang-Mills and Seiberg-Witten equations. These equations involve geometric quantities such as connections and spinors, and the equations themselves depend on parameters such as a Riemannian metric on a manifold. The basic technique is to artfully count the solutions of these equations, and then prove that the count is an invariant of the underlying manifold, ie is independent of any extra parameters. This idea, introduced by Donaldson, has led to remarkable advances in our understanding of low-dimensional topology.

Gauge theory with parameters seeks to carry out a similar scheme to define invariants for families of metrics on a given manifold. I will explain how 1-parameter gauge theory can detect interesting geometric and topological properties of certain 4-manifolds. For example, this theory shows that homotopy of diffeomorphisms does not imply isotopy in dimension 4. Put another way, we find components of the diffeomorphism group of some simple manifolds that are not detected by more traditional methods. In a more geometric vein, I will explain a similar method to show that the space of metrics of positive curvature on a 4-manifold is often disconnected.

This talk should be accessible to graduate students.

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