Kolmogorov complexity and Martin-Lof randomness

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(Tea 3:10 pm Math Lounge)

Abstract

Kolmogorov defined the complexity of a finite binary string to be the length of the shortest program that generates it. Most strings have Kolmogorov complexity close to their lengths; in other words, they cannot be compressed. Such strings are considered to have high information content. Martin-Lof, a student of Kolmogorov, considered the complexity of infinite binary sequences. He defined a sequence to be random if it passes certain effectively presented statistical tests. Levin and Chaitin both modified Kolmogorov complexity to characterize Martin-Lof randomness, while Schnorr gave a characterization in terms of effective betting games. Thus we have three equivalent formulations of randomness arising from three different fundamental intuitions: random sequences are "unremarkable", "incompressible" and "unpredictable".

This talk will focus on some of the main themes of my work on Kolmogorov complexity and Martin-Lof randomness. We will examine the complexity of initial segments of random reals, giving new insight into the behavior of this complexity and applying it to study the "degrees of randomness" of infinite sequences. Along the way, we are able for the first time to characterize Martin-Lof randomness in terms of Kolmogorov's original complexity measure. We will also contrast the notions of "information" found in computability theory and algorithmic randomness. For example, we will see that an appropriate upper bound on the computational power of a random sequence actually enforces a lower bound on its degree of randomness. Similarly, there are connections between high initial segment complexity and low computational power.

This talk should be accessible to graduate students.