Recent developments in group theory

Mauricio Gutierrez
Tufts University

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L02 Carson Hall, 4:00 pm
(Tea 3:30 pm Math Lounge)

Abstract

Starting in the early 80's with the work of Gromov, Cannon and Thurston, Group Theory has experienced a complete revolution. Gone is the classical Combinatorial Group Theory that viewed groups as quotients of free groups and the more recent Topological Group Theory that viewed groups as fundamental groups of spaces.

In part this was so because of the failure of either theory to grapple with the classical problems posed by Max Dehn in 1912: the word problem, the conjugacy problem and the isomorphism problem. These were found to be insoluble (in general) in the 1950's by Novikov and Boone and to be (in particular) soluble in a minuscule number of cases (groups with a single relation).

Instead one views groups as acting as a group of isometries on metric spaces that admit geodesics, that is if x and y are two points on a metric space X at a distance d from one another, there is an isometry $g : [0, d] \to X$ with $g(0) = x$ and $g(d) = y$.

Gromov, based on the work of A. Alexandrov (work that was nearly unknown in the west), defined a notion of curvature on these spaces. Then he was able to show that if a group acts geometrically (by isometries, properly discontinuously and with compact quotient) on a space of non-positive curvature, then its word problem and its conjugacy problem are solvable in quadratic time. Furthermore if the space is hyperbolic (thin geodesic triangles) then one can solve the problems in linear time.

Many groups turn out to act geometrically on non-positively curved spaces; for example, all Coxeter groups, surface groups and most 3-manifold groups. The latter two were studied by Nielsen in the 20's and many of his techniques were revived by Gromov and Cannon.

The talk should be accessible to graduate students, and to those undergrads who are familiar with fundamental groups and covering spaces.

This talk should be accessible to undergraduates.