Combinatorics of Elliptic Curves and Chip-Firing Games

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Abstract

For a given elliptic curve E over a finite field F_q, we let $N_k = \#E(F_{q^k})$, where F_{q^k} is a kth degree extension of the finite field F_q. Because the Zeta Function for E only depends on q and N_1, the sequence $\{N_k\}$ only depends on those numbers as well. More specifically, we observe that these bivariate expressions for N_k are in fact polynomials with integer coefficients, which alternate in sign with respect to the power of N_1.

This motivated a search for a combinatorial interpretation of these coefficients, and one such interpretation involves spanning trees of a certain family of graphs. In this talk, I will describe this combinatorial interpretation, as well as applications and directions for future research. This will include determinantal formulas for N_k, factorizations of N_k, and the definition of a new sequence of polynomials, which we call elliptic cyclotomic polynomials.

One of the important features of elliptic curves which makes them the focus of contemporary research is that they admit a group structure. During the remainder of this talk I will describe chip-firing games, how they provide a group structure on the set of spanning trees, and numerous ways that these groups are analogous to those of elliptic curves.

This talk should be accessible to graduate students.