Categorical Sequences

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007 Kemeny Hall, 4:00 pm

(Tea 3:30 pm 300 Kemeny Hall)

Abstract

A Lusternik-Schnirelmann cover (L-S cover for short) of a subspace A of X is a finite cover

$$A \subseteq U_0 \cup U_1 \cup \cdots \cup U_k$$

where each U_i is open in X and each inclusion map U_i into X is homotopic to a constant map. The Lusternik-Schnirelmann category of A in X is the least k for which an L-S cover exists, or ∞ if there is no such cover; the notation is $\mathrm{cat}_X(A) = k$.

If X is a CW complex, then we can look at the numbers

$$\mathrm{cat}_X(X_0), \mathrm{cat}_X(X_1), \ldots, \mathrm{cat}_X(X_n), \mathrm{cat}_X(X_{n+1}), \ldots$$

which form a weakly increasing sequence. Interestingly, this sequence is independent of the choice of CW decomposition of X, and so it is an invariant of the homotopy type of X.

I will talk about some surprising numerical structure that can be found in this sequence, show how it can be used to prove interesting results, and speculate about further interesting problems.

Some of this was discovered while I was a JWY Instructor at Dartmouth, and some of it was joint work with Nick Scoville and Rob Nendorf. The talk should be comprehensible to anyone familiar with homotopy of maps.

This talk should be accessible to graduate students.