

Nilpotent Groups, Subelliptic Operators, and Index Theory

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Abstract

The Atiyah-Singer index formula expresses the Fredholm index of an elliptic operator on a closed manifold as a function of a cohomology class associated to the operator. On the face of it, this cohomology class is defined if and only if the operator is elliptic. However, developing ideas from noncommutative geometry, we show that for a natural class of *sub*-elliptic operators (which are Fredholm, but not elliptic) the Atiyah-Singer formula is still valid. The difficulty is to construct an appropriate cohomology class for such operators that can be plugged into the index formula. While the machinery used to prove that the Atiyah-Singer formula, applied to this novel cohomology class, is still valid in the subelliptic case is quite formidable, the result is surprisingly simple and elegant, and can be explained without reference to the underlying noncommutative techniques.