Appendix J

Errata

This list of errata is maintained by the author. If you notice additional instances that are incorrect, misleading or poorly explained, please notify the author at `dana.williams@dartmouth.edu`. This page was last updated on July 19, 2021 at 13:39.

Page 2, Definition 1.1: It would have sufficed to assume that topological groups satisfy the T_0-axiom of separability; that is, the distinct points have distinct closures. In a topological group, this forces points to be closed. The proof is similar to that of Lemma 1.13. Let $s \neq r$ be points in G. We may as well assume $s = e$ and that we have a neighborhood U of e not meeting r. Continuity of multiplication implies there is a neighborhood V of e such that $V^2 \subset U$. Then $\overline{V} \subset U$ by Lemma 1.12.

Page 5, Definition 1.16: It turns out, very few of the authorities agree on what a locally compact space is if the space is not Hausdorff. The definition I have given seems the most natural to me as these are the sorts of spaces that arise in the study of C^*-algebras. Nevertheless, I should have pointed out that many authors define a space to be locally compact if every point has a compact neighborhood. One drawback of my definition is that one can have compact spaces that are not locally compact.

Page 7, Lemma 1.30: The "$\Longleftarrow$" implication in the lemma is false as stated. The last if and only if should be replaced by "If $f_n \to f$ in $C(X,Y)$, then whenever $x_n \to x$ in X we have $f_n(x_n) \to f(x)$. Conversely, if every subnet $\{ f_{n_j} \}$ has the property that $x_{n_j} \to x$ in X implies $f_{n_j}(x_{n_j}) \to f(x)$, then $f_n \to f$ in $C(X,Y)$." This is actually what is proved on page 8.

Page 30, Line −10: Replace "a compact convex set" with "a compact set".

Page 54, Line 19: Replace "$*$-subalgebra of $M(A \rtimes_\alpha G)$" with "$*$-subalgebra of $A \rtimes_\alpha G$".

Page 54, Line −10: Replace "$a, b \in A$" with "$a, b \in A \rtimes_\alpha G$".

Page 63, Line −1: Delete "on the previous page".

Page 70, Equation (2.37): There is a factor of $\frac{1}{2n+1}$ missing in the last two formulas:

$$E_n(v^k * u^m) = \frac{1}{2n+1} \sum_{j=-n}^{j=n} v^j * v^k * u^m * v^{-j} = \frac{v^k * u^m}{2n+1} \sum_{j=-n}^{j=n} \rho^{jm}.$$

Page 70, line −10: The is a missing factor of $\frac{1}{2n+1}$ is the limit in the displayed equation:

$$\lim_{n\to\infty} E_n(v^k * u^m) = v^k * u^m \lim_{n\to\infty} \frac{1}{2n+1} \sum_{j=-n}^{j=n} \rho^{jm} = 0.$$

Page 71, line −6: Both occurrences of "$\theta' = \theta \mod 1$" should be "$\theta' = \pm\theta \mod 1$".

Page 72, line 3 of Remark 2.62: "Propositions" should be "Proposition".

Page 76, line −5: "subtly" should be "subtlety".

Page 79, line −5: At least one expert has noted that the word "current" suggests that the "Danish" notation is *the* standard notation. There are others; for example, the notation $C^*(G, A, \alpha)$ is also used. I guess my prejudice is clear.

Page 88, Lines 8–9: The phrase "Since f is uniformly continuous, the triangle inequality implies" is not very helpful since f is only defined on N. We could "rescue" it by noting that f is the restriction to N of some F in $C_c(G, A)$ by Lemma 8.54 on page 258 and appealing to the uniform continuity of F. (Of course, I can't, with a straight face, claim that this forward reference is what I originally intended.) Alternatively, we can establish the existence of V'' with the required property via a compactness argument as follows. We can assume that V' is precompact and symmetric. If no V'' exists, then for each $V \subset V'$ we can find $h_V \in V$ and $n_V \in N$ such that

$$\|f(h_V^{-1} n_V h_V) - f(n_V)\| \geq \frac{\epsilon}{3}.$$

Since we must have $n_V \in V'(\operatorname{supp} f)V'$ and since $V'(\operatorname{supp} f)V'$ has compact closure, we can pass to a subnet, relabel, and assume that $n_V \to n$ while $h_V \to e$ This leads to a contradiction and establishes the existence of V'' as claimed.

Page 91, Line −6: Replace "Assume that N and K" by "Assume that N and H".

Page 95, line 11: Replace "$x \in G_x$" with "$s \in G_x$".

Page 110, Equation 4.1: Replace "h" by "s".

Page 129, line 9: Replace "$C_0(X)$" by "$C_0(G\backslash X)$".

Page 153, Footnote 3: Although the footnote is correct in the case G is second countable and $\mathcal{H}$ is separable, it is unfortunately *not* proved in Section 9.3 on page 283 as claimed. I have included a proof in Note J.3 below. In the general case, I have not completely sorted out the details, but it will certainly be necessary to work with separably valued (or essentially separably) valued functions and locally μ_G-almost everywhere equivalence classes.

Page 177, second paragraph of (c) $\Longrightarrow$ (d): To see that $G\cdot x$ has a dense open Hausdorff subset, it is not sufficient to appeal directly to Lemma 6.3. Instead, notice that *every* nonempty subset S of X has a relatively open nonempty Hausdorff subset: by Lemma 6.3, $\overline{S}$ has a dense open Hausdorff subset U and U must have nontrivial intersection with S. But then a maximal such open set must be dense by exactly the same argument as in (b) $\Longrightarrow$ (c) in the proof of Lemma 6.3 — the fact that F is closed is never used.[1]

Page 195, line 1: Replace "$\widehat{R}$" with "$\widehat{\mathbf{R}}$".

Page 227, line 7: Replace "principle" by "principal".

Page 232, line 9: Proposition 8.7 on page 232 can be easily strengthened. See Note J.6 on page 536.

Page 234, line −4: Corollary 8.9 on page 234 can be easily strengthened. See Note J.6 on page 536.

Page 240, lines 11–13: The assertion that separable regular systems are EH-regular is true, but does not follow easily from Theorem 8.16 on page 237. (Unfortunately, in Definition 8.10 on page 235, a irreducible representation can be induced from a stability group without its kernel being an induced primitive ideal! In the first case, the restriction merely must contain P and in the later it must equal P.) However, the statement in the text is true; this follows, for example, from [66, Proposition 20]. A simple modification of Theorem 8.16 on page 237, which also implies the EH-regularity assertion, is given in Note J.6 on page 536.

Page 240, line 14: Replace "ones were" by "ones where".

Page 241, Line 15: Replace "equivalent to" by "equal to".

Page 242, Remark 8.23: Archbold & Spielberg have a tidy proof of a more general result in their 1994 paper in volume 37 of the *Proceedings of the Edinburgh Mathematical Society.*

[1] Alternatively, we can forget about showing every set has a dense open Hausdorff subset, and just notice that since open subsets of Baire spaces are Baire by [109, Lemma 48.4], it suffices in the original proof to just note that $G \cdot x$ has a nonempty open Hausdorff subset.

Page 252, Lemma 8.38: The "only if" portion of the lemma is incorrect as stated. The last sentence should be replaced with "If the net $\{ I(F_j) \}$ converges to $I(F)$ in $\mathcal{I}(A)$, then given $P \in F$ there is a subnet $\{ I(F_{j_k}) \}$ and $P_k \in F_{j_k}$ such that $P_k \to P$ in $\operatorname{Prim} A$. If every subnet of $\{ I(F_j) \}$ has a subnet $\{ I(F_{j_{k_i}}) \}$ with $P_i \in I(F_{j_{k_i}})$ such that $P_i \to P$ in $\operatorname{Prim} A$, then $\{ I(F_j) \}$ converges to $I(F)$ in $\mathcal{I}(A)$." This is what is actually proved in the text.

Page 255, line 7: Replace "points" by "point".

Page 256, line −4: Replace "are" by "is".

Page 259, line 16: Replace "$\|\kappa_H(f)\|_1$" by "$\|\kappa_H(f)\|_{A \rtimes H}$".

Page 262, line 11: Replace "not be" by "not to".

Page 275, line 16: Replace "$s \cdot \tilde{r}_{s \cdot Q}$" with "$s \cdot \tilde{r}_Q$".

Page 284, Lemma 9.16: I should have pointed out that we need $\mathcal{H}$ separable in order to know that the integrand of equation (9.27) is a Borel function of (P, s).

Page 286, Lines 15–16: The statement "Notice that each ... in the obvious way" requires that we know that $\mathcal{K}(P)$ can be realized as the Hilbert space of β^P-square integrable Borel functions on G which transform in the appropriate way. This was not proved in Proposition 5.4 on page 153, nor as claimed in footnote 3 on page 153, is a proof given in Section 9.3 on page 283 (see the errata for page 153 above). Instead, the result must be proved directly, and I have done so in Note J.3 below.

Page 331, line −9: Replace "where" by "were".

Page 333, Definition B.5: Of course, "separately valued" should be "separably valued" here and elsewhere.

Page 344, line 25: Replace "Hill and Phillips" by "Hille and Phillips".

Page 345, line 8: Replace "vanishing of" by "vanishing off".

Page 356, line 7: Replace "$\ker \rho \subset J$" with "$\ker \rho \supset J$".

Page 338, line 7: Replace "B-measurable" with "measurable".

Page 358, line 19: Replace "$\big(f(x)g - fg\big) \cdot a$" with "$\big(f(x)g - fg\big) \cdot b$".

Page 359, line 9: Replace "$m(x)b(x)$" with "$m(x)b$".

Page 367, Theorem C.26: Item (c) is awkwardly stated (at best). The point is that if either (a) (or) (b) hold, then A is a $C_0(X)$-algebra so that "$C_0(X)$-linear" has a meaning. Of course, if A is imply a C^*-algebra which is isomorphic to the section algebra, then it is premature to talk about a $C_0(X)$-linear isomorphism. Of course, it inherits a $C_0(X)$-algebra structure in this case and then the isomorphism is $C_0(X)$-linear by definition.

Page 389, Corollary D.34 The corollary is correct as stated. But in the proof it should be noted that d_0 is also a Radon-Nikodym for $\sigma_*(\mu_G \times \mu)$ with respect to $\mu_G \times \mu$. Hence part (a) is valid.

Page 410, Line 21: Lemma D.20 does not suffice to establish Remark F.3 since the lemma assumes that the functions which separate points are **C**-valued. Here π is X-valued. But a variation on the proof of Lemma D.20 will suffice.

Page 418, Footnote 5: Delete "; see page 423".

Page 418, line 13: Replace "an appeal" with "and appeal".

Page 419, line −11: Replace "decomposition of ρ" with "decomposition of ρ with respect to $\mathcal{C}$".

Page 427, Line −9: Delete "In fact, it suffices to take f and g in a fundamental sequence." It does not: consider a trivial bundle and the fundamental sequence given by constant functions.

Page 452, line 4: It is not correct to say that $(\operatorname{Prim} A \times \operatorname{Prim} A, \gamma)$ is a measured groupoid. The groupoid $\mathcal{G} := \operatorname{Prim} A \times \operatorname{Prim} A$ should be replaced by the equivalence relation $\mathcal{R} \subset \operatorname{Prim} A \times \operatorname{Prim} A$ where $P \sim Q$ if there is a $s \in G$ such that $Q = s \cdot P$. Since $\mathcal{R}$ is the continuous image, under the map $(P, s) \mapsto (P, s \cdot P)$, of the Polish space $\operatorname{Prim} A \times G$, $\mathcal{R}$ is certainly analytic. In fact, if $K \subset G$ is compact, then it is easy to see that the image of $\operatorname{Prim} A \times K$ is closed. Since G is σ-compact, $\mathcal{R}$ is a F_σ in $\operatorname{Prim} A \times \operatorname{Prim} A$, and therefore Borel — and hence standard. Certainly, there is no harm in viewing γ as a σ-finite measure on $\mathcal{R}$, and Lemma 9.1 shows that γ is equivalent to γ^{-1}. We can let λ^P be the measure on $\mathcal{R}$ supported on $\{ P \} \times G \cdot P$ given by the measure β_P described in Remark 9.4:

$$\lambda^P(f) := \int_{\operatorname{Prim} A} f(P, Q)\, d\beta_P(Q).$$

Then

$$\begin{aligned}\gamma(f) = \int_{\mathcal{R}^{(0)}} \int_{\mathcal{R}} f(P, Q)\, d\lambda^P(P, Q)\, d\mu(P) \\ = \int_{\operatorname{Prim} A} \int_{\operatorname{Prim} A} f(P, Q)\, d\beta_P(Q)\, d\mu(P).\end{aligned}$$

Since $\beta_P = \beta_{s \cdot P}$ by Lemma 9.3 and Remark 9.4, we have $(Q, P) \cdot \lambda^P = \lambda^Q$ for all $(Q, P) \in \mathcal{R}$. Therefore if $[\gamma]$ is the measure class corresponding to γ (as a measure on $\mathcal{R}$), then $(\mathcal{R}, [\gamma])$ is a measured groupoid *a la* Ramsay (see [108, Definition 4.1]), and the proof of Lemma 9.2 proceeds with $\mathcal{R}$ in place of $\mathcal{G}$.

Page 454, Lemma H.2 The lemma should be reworded to say that if $F_i \to F$ in $\mathscr{C}(X)$, then assertions (a) and (b) hold. However, to see that $F_i \to F$, then we need to show that *every subnet* of $\{ F_i \}$ has properties (a) and (b).

Page 454, line −9: Replace "$F_i \cap U$" with "$F_i \cap U_3$".

Page 463, Footnote 3, Line 3: Replace "a subset has a" by "a pre-compact subset has a".

Page 464, Line −8: Replace "for μ_G-almost all" to "for locally μ_G-almost all". (Alternatively, we could assume that G is second countable.)

Page 465, Line 4: Replace "μ_G-almost everywhere" with "locally μ_G-almost everywhere".

Page 490, Proof of Lemma I.11: The statement on line 17 — that "φ' is *clearly* normal" — is hard to justify. I have included a proof of this in Note J.1 below. As an alternative, I have also supplied a different proof of the lemma in Note J.2 on the facing page.

Page 503, line 13: Delete the comma after "standard Borel spaces".

Page 525, Reference [108]: Replace "Texas Tech" with "Texas Christian University".

Note J.1. We want to show that if (X, μ) and (Y, ν) are standard measure spaces and if $\tau : X \to Y$ is a Borel map such that for every ν-null set N, $\tau^{-1}(N)$ is μ-null, then map $\varphi : L^\infty(Y, \nu) \to L^\infty(X, \mu)$ defined by $\varphi(f)(x) := f\big(\tau(x)\big)$ is normal.[2] (The condition on τ and null sets is required so that φ is well defined on almost-everywhere equivalence classes.) Here we are viewing $L^\infty(Y, \nu)$ as a von Neumann algebra of operators on $L^2(Y, \nu)$, which is a *separable* Hilbert space (Lemma D.41 and Definition I.33).

So, suppose that $\mathcal{F}$ is a filtering subset of $L^\infty(Y, \nu)^+$ with $f = \operatorname{lub}(\mathcal{F})$. By [29, Appendix II], f is in the strong closure of $\mathcal{F}$. Since the strong operator topology is metrizable on bounded subsets (because $L^2(Y, \nu)$ is separable), we can find a *sequence* $\{ f_i \}$ in $\mathcal{F}$ such that $f_i \to f$ in the strong operator topology. After changing each f_i on a null set, we can also assume that for each i, $f_i(y) \le f(y)$ for all $y \in Y$. Since ν is a finite measure, $1 \in L^2(Y, \nu)$ and we must have

$$\int_Y f_i(y)\, d\nu(y) \to \int_Y f(y)\, d\nu(y).$$

Thus,

$$\lim_i \int_Y \big(f(y) - f_i(y)\big)\, d\nu(y) = \int_Y |f(y) - f_i(y)|\, d\nu(y) = 0.$$

Thus, we can pass to a subsequence, relabel, and assume that there is a ν-null set N such that $f_i(y) \to f(y)$ for all $y \notin N$. But then $\varphi(f_i)(x) \to \varphi(f)(x)$ for all $x \notin \tau^{-1}(N)$. Since the latter is a μ-null set, it follows that

$$\varphi(f) = \operatorname{lub}\{ \varphi(g) : g \in \mathcal{F} \}.$$

Thus, φ is normal as claimed. $\square$

[2] The definition of a normal map is given on page 502.

Note J.2. Here we give a different proof of Lemma I.11. Let $\mathcal{A}$ be the range of φ. By Theorem I.10 on page 488, we can realize $\mathcal{A}$ with the image of those Borel functions on X which are constant on equivalence classes for a smooth equivalence relation on X. Furthermore, $\mathcal{A}$ is isomorphic to $L^\infty(Z,\rho)$ where $Z = X/\!\sim$ and $\rho = q_*\mu$ (where $q : X \to Z$ is the quotient map). Since φ gives us an isomorphism of $L^\infty(Y,\nu)$ onto $L^\infty(Z,\rho)$, Corollary I.38 on page 502 implies that there is a Borel map $\tilde{\tau} : Z \to Y$ such that $\varphi(f)([x]) = f\big(\tilde{\tau}([x])\big)$. (We can extend $\tilde{\tau}$ from Corollary I.38 on the null set N in any way we like.) Then, assuming $\tau := \tilde{\tau} \circ q$, we have $\varphi(f)(x) = f\big(\tau(x)\big)$.

Note J.3. In this note, I want to show that the Hilbert space $\mathcal{V}$ in Proposition 5.4 is *naturally* isomorphic to the Hilbert space $L^2_u(G,\mu_{G/H},\mathcal{H})$ built from functions on G as described in footnote 3 on page 153. I will use the notations and set up from Proposition 5.4 on page 153. *However, I will assume throughout that G is second countable and $\mathcal{H}$ is separable* (see Remark J.5 on the next page for comments on these hypotheses). Specifically, $L^2_u(G,\mu_{G/H},\mathcal{H})$ is the set of μ_G-almost everywhere equivalence classes of functions in $\mathcal{L}^2_u(G,\mu_{G/H},\mathcal{H})$, where the later consists of Borel functions $\xi : G \to \mathcal{H}$ such that $\xi(st) = u_t^{-1}\big(f(s)\big)$ for *all* $s \in G$ and $t \in H$, and such that $sH \mapsto \|f(s)\|$ is in $L^2(G/H,\mu_{G/H})$. As in the proof of Lemma 9.16 on page 284, it is not hard to see that

$$(\xi \mid \eta) := \int_{G/H} \big(\xi(r) \mid \eta(r)\big)\, d\mu_{G/H}(rH)$$

is an inner product on $L^2_u(G,\mu_{G/H},\mathcal{H})$,[3] and the proof of Proposition 9.18 on page 286 shows that $L^2_u(G,\mu_{G/H},\mathcal{H})$ is a Hilbert space. Furthermore, $\mathcal{V}_c$ is clearly a subspace of $\mathcal{L}^2_u(G,\mu_{G/H},\mathcal{H})$, and it is immediate that we can view $\mathcal{V}$ as a subspace of $L^2_u(G,\mu_{G/H},\mathcal{H})$. What we want is the following.

Lemma J.4. *The subspace $\mathcal{V}_c$ is dense in $L^2_u(G,\mu_{G/H},\mathcal{H})$. Consequently, we can identify $\mathcal{V}$ and $L^2_u(G,\mu_{G/H},\mathcal{H})$.*

Proof. We recall from the proof of Proposition 5.4 on page 153 that for each $f \in C_c(G,A)$ and $h \in \mathcal{H}$, we have $W(f\otimes h) \in \mathcal{V}_c$ and that the image $\mathscr{M}$ of $C_c(G,A)\odot\mathcal{H}$ under W is dense in $\mathcal{V}_c$. (The map W is defined at the beginning of the proof of Proposition 5.4.) Therefore, it will suffice to see that if $\xi \in \mathcal{L}^2_u(G,\mu_{G/H},\mathcal{H}) \cap \mathscr{M}^\perp$, then ξ is zero μ_G-almost everywhere. Note that, since (π,u) is covariant,

$$\pi\big(\alpha_r^{-1}\big(f(rt)\big)\big)u_t = u_t\pi\big(\alpha_{rt}^{-1}\big(f(rt)\big)\big).$$

Consequently,

$$\begin{aligned}\big(W(f\otimes h) \mid \xi\big) &= \int_{G/H} \big(W(f\otimes h)(t) \mid \xi(r)\big)\, d\mu_{G/H}(rH)\\ &= \int_{G/H}\int_H \big(\pi\big(\alpha_{rt}^{-1}\big(f(rt)\big)\big)h \mid \xi(rt)\rho(rt)^{-\frac{1}{2}}\, d\mu_H(t)\, d\mu_{G/H}(rH)\end{aligned}$$

[3] We are using the separability of $\mathcal{H}$ here — see the end of Remark J.5 on the next page.

which, by Proposition H.11 on page 462, is

$$= \int_G \big(\pi\big(\alpha_r^{-1}\big(f(r)\big)\big)h \mid \xi(r)\big)\rho(r)^{\frac{1}{2}}\, d\mu_G(r).$$

Define $N : C_c(G, A) \odot \mathcal{H} \to C_c(G, \mathcal{H})$ by

$$N(f \otimes h)(r) := \rho(r)^{\frac{1}{2}}\pi\big(\alpha_r^{-1}\big(f(r)\big)\big)h.$$

Let $\mathcal{A}$ be the image of N. Now, invoking the obvious analogue of Lemma H.15 on page 464 (with $C_c(G)$ replaced by $C_c(G, \mathcal{H})$), we see that it suffices to show that $\mathcal{A}$ is dense in $C_c(G, \mathcal{H})$ in the inductive limit topology. Using a partition of unity argument, it will suffice to show that for each $r \in G$ and $\epsilon > 0$ there is a $f \in C_c(G, A)$ such that

$$\|N(f \otimes h)(r) - h\| < \epsilon.$$

But this is an easy consequence of the nondegeneracy of π. □

Remark J.5 (The Separability Hypotheses). I dislike invoking separability hypotheses unnecessarily, but the price was too high in Lemma J.4 on the preceding page. Forcing G to be second countable means that locally μ_G-null sets are null. This has a number of comforting consequences not the least of which is that we can apply the results from Section H.2 on page 456 — specifically Proposition H.11 on page 462, Lemma H.14 on page 463 and Lemma H.15 on page 464— as is. (Of course, nonseparable analogues of these results can be found in places like [54, Chap. III §14], but there is a price to be paid.) Insisting that $\mathcal{H}$ is separable means that we can work with the results in Section I.4 on page 490 without worrying whether our functions are separably valued or essentially separably valued (see the definitions of measurablity in Appendix B.1 on page 331). For example, we need ξ and η to be separably valued to conclude that

$$s \mapsto \big(\xi(s) \mid \eta(s)\big)$$

is Borel when ξ and η are. (This was, unfortunately, not mentioned in Lemma 9.16 on page 284.)

Note J.6. It is possible to easily give a sharpening of Theorem 8.16 on page 237 that could be useful. This requires strengthening of Proposition 8.7 on page 232 and Corollary 8.9 on page 234 which are also of potential interest. The sharpening of Theorem 8.16 is that we can add the following line to the end of its statement:

> In particular, if ρ is an irreducible representation of $A \rtimes_\alpha G$, then there is a primitive ideal $P \in \operatorname{Prim} A$ and an irreducible representation L of $A \rtimes_{\alpha|_{G_P}} G_P$ such that $\operatorname{Res}(\ker L) = P$ and $\operatorname{Ind}_{G_P}^G L$ is equivalent to ρ.

To see this, the key observation is to observe that the proof of Proposition 8.7 can be modified to show that the hypotheses that (A, G, α) is regular and that $G\backslash \operatorname{Prim} A$ is almost Hausdorff imply that points are closed in $\operatorname{Prim} A$. (This is just a fancy way of saying that A/P is simple for each $P \in \operatorname{Prim} A$.) To see that this is the case, notice that on page 232, line -16, $A(G \cdot P)/Q$ must be simple since $\operatorname{Prim} A(G \cdot P)$ is Hausdorff. But $A(G\cdot)/Q$ is isomorphic to A/P. Having taken care of Proposition 8.7, Corollary 8.9 on page 234 can be easily modified by replacing "with $\operatorname{Res}(\ker L) \supset P$" by "with $\operatorname{Res}(\ker L) = P$". Having fixed both these preliminary results, in the proof of Theorem 8.16 we make the following modifications on page 239:

- In line 6, replace "with $\operatorname{Res}(\ker L) \supset K$" by "with $\operatorname{Res}(\ker L) = K$".
- In line 10, replace "with $\operatorname{Res}(\ker L') \supset Q$" by "with $\operatorname{Res}(\ker L') = Q$".
- In line 18, replace "Since $\operatorname{Res}(\ker \overline{L'}) \supset P$" by "Since $\operatorname{Res}(\ker \overline{L'}) = P$".