Math 32, 'The Shape of Space'

Peter Doyle

Winter 2022

Welcome to 'The Shape of Space'!

In this course we will cover a broad range of topics in 'geometry', which we take to include such fields as topology and differential geometry as well as more classical geometry. The aim will be to convey the richness, diversity, connectedness, depth and pleasure of mathematics.

This course is a descendant of a course called 'Geometry and the Imagination', developed about 30 years ago at Princeton by Bill Thurston, Peter Doyle, John Conway, and Jane Gilman. That title was cribbed from the English translation of the famous book 'Anschauliche Geometrie' by Hilbert and Cohn-Vossen, which literally means something more like 'visual geometry'. Math 32 at Dartmouth is called 'The Shape of Space' in honor of Bill Thurston's student Jeff Weeks, Dartmouth '78, who wrote a book with that name, used as one of our course textbooks. While 'imagination' may no longer figure in the course title, it is a main emphasis of our course (and Jeff's book!). *Imagination*, an essential part of mathematics, means not only the facility which is imaginative, but also the facility which calls to mind and manipulates mental images. One aim of the course is to develop the imagination.

While the mathematical content of the course will be high, we will try to make it as independent of prior background as possible. Abstract algebra, for example, is not a prerequisite. While you won't need a heavy formal background for the course, you do need a commitment of time and energy, and a dose of 'mathematical maturity'.

Philosophy

In this course, we will emphasize the *process* of thinking about mathematics. Assignments will involve thinking and writing, not just grinding through formulas. There will be a strong emphasis on projects and discussions rather

than lectures. All students are expected to get involved in discussions, within class and without.

The spirit of mathematics is not captured by spending 3 hours solving 20 look-alike homework problems. Mathematics is thinking, comparing, analyzing, inventing, and understanding. The main point is not quantity or speed—the main point is quality of thought. The goal is to reach a more complete and a better understanding. We will use materials such as Polydrons, straws, mirrors, scissors, and tissue paper not because we think this is easier than solving algebraic equations and differential equations, but because we think that this is the way to bring thinking and reasoning to the course.

Instructor

Peter Doyle, 331 Kemeny Hall.

Class meetings

The class meets in the 2A slot, 2:25–4:15 in 007 Kemeny Hall. We will be using the X-hour, F 4:35–5:25 (Yikes!). Keep this time open!

When you will not able to attend class, I would appreciate it if you would send me email in advance.

Course materials and textbooks

The main resources for the course will be the handouts for 'Geometry and the Imagination' on my webpage; 'The Shape of Space, Third Edition', by Jeffrey R. Weeks (the second edition will also do), and 'The Symmetries of Things', by John Conway. (The Conway book will be lightly used, and you may be able to make do without purchasing it, but it is a beautiful and inspirational book.)

Components of the course

The major components of the course are the course journal and a major final project. There will be no exams, though there will be a couple of quizzes.

Journals I will ask you to keep a journal for the course, in which to write about a wide variety of topics, including some number of specific assignments. Journal entries are to be submitted electronically through Canvas.

You may consult any source in completing the assignments, as long as you write up the solutions yourself, and cite any source you've drawn upon. This would include indicating that you worked with a classmate. You don't need to cite me; the course texts; the Geometry and the Imagination notes; any other source that has been specifically suggested; or fleeting conversations with classmates. The idea here is to know where you're coming from.

Quizzes There will be a couple of quizzes, to make sure you have mastered basic material and techniques. The main focus of the course lies elsewhere.

Projects

Course projects will be presented during class at the end of the term (1–8 March). Preliminary versions will be due as follows. Take 1: 10 February; Take 2: 17 February; Take 3: 24 February.

Due to the size of the class, barring exceptional circustances, students will be expected to work in pairs. Larger groups are not permitted.

Your project may be on any topic related to geometry or topology. It should be something you are really interested in, because a good final project will require a minimum of 17 Keplers of effort.

Grading

Grades will be subjective, based on my assessment of what students have put into and gotten out of the course.

Covid

We can expect bumps on the road this quarter. Due to the flexible nature of this course, we should be able to handle them.

Disabilities

Students with disabilities who may need disability-related academic adjustments and services for this course are encouraged to see me privately as early in the term as possible. Students requiring disability-related academic adjustments and services must consult the Student Accessibility Services office (Carson Hall, Suite 125, 646-9900). Once SAS has authorized services, students must show the originally signed SAS Services and Consent Form and/or a letter on SAS letterhead to their professor. As a first step, if students have questions about whether they qualify to receive academic

adjustments and services, they should contact the SAS office. All inquiries and discussions will remain confidential.

Religious observances

Some students may wish to take part in religious observances that occur during the academic term. Any religious observation that conflicts with participation in the course should be discussed with the instructor in advance. If further support is required, please contact the Tucker Center (tucker.center at dartmouth.edu).

Sources

- The general introduction and 'philosophy' sections are adapted from materials from the original 'Geometry and the Imagination' course.