

Ed Pegg, Jr. has identified the ‘Doyle graph’ as the two-cycle graph  $\{13, 7, 4\}^9$ , thus exhibiting it as the union of two Hamiltonian circuits. (See Figure 1.) To check that there is a graph automorphism exchanging these two cycles we compute (modulo 27):

$$13 + 7 + 4 + 13 + 7 + 4 + 13 + 7 + 4 + 13 + 7 + 4 + 13 = 4 \cdot (-3) + 13 = 1;$$

$$7 + 4 + 13 + 7 + 4 + 13 + 7 = 2 \cdot (-3) + 7 = 1;$$

$$4 + 13 + 7 + 4 = 1 \cdot (-3) + 4 = 1.$$

But it’s more fun to trace this out by hand!

Thinking of this graph as arising from the Cayley graph of the presentation

$$\langle a, c \mid a^9 = c^9 = 1, c^3 = a^{-3}, a^3 = c^{-3}, c^{-1}ac = a^4, a^{-1}ca = c^4 \rangle,$$

these cycles may be represented as  $(aca)^9$  and  $(cac)^9$ . Exercise: Using this information, turn Figure 1 into a bona fide Cayley graph by coloring its edges. (The orientations are all to be taken counter-clockwise.)

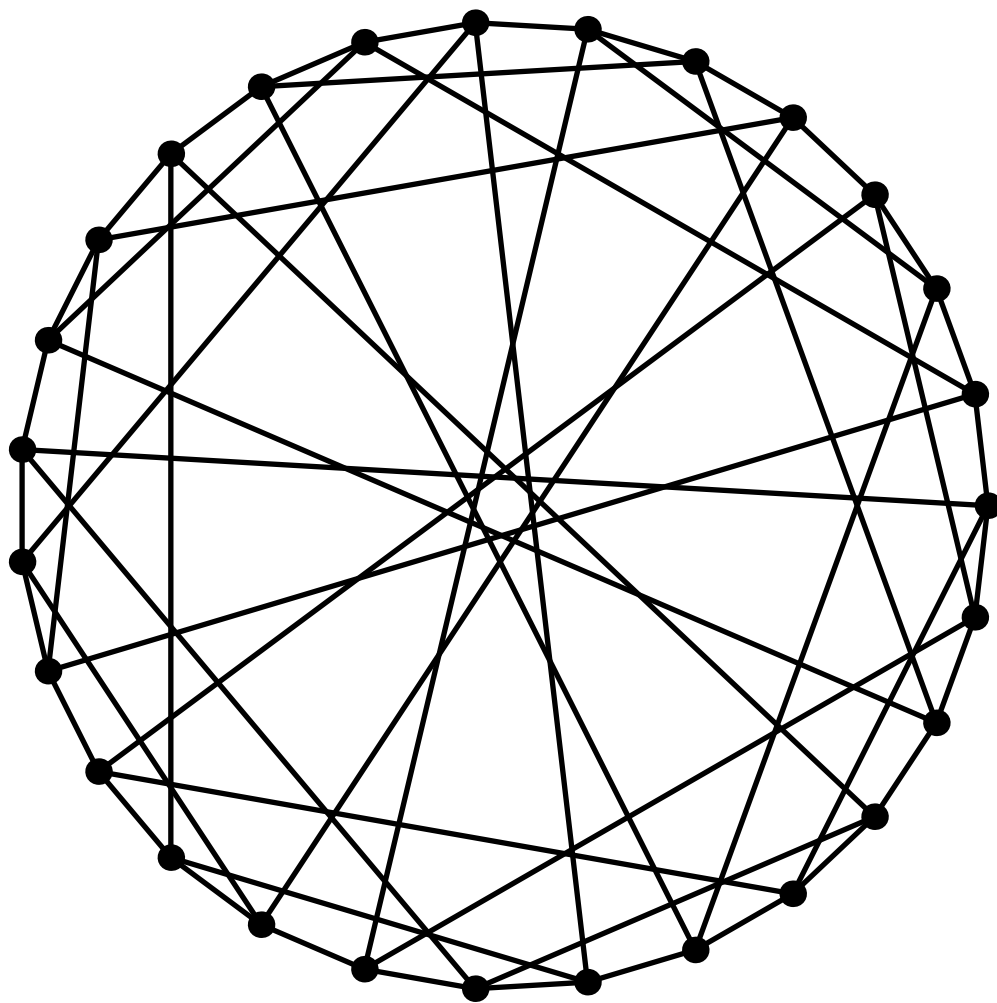


Figure 1: The 'Doyle graph' as the two-cycle graph  $\{13, 7, 4\}^9$ .