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April 19, 1976

Mr. P. Doyle
2 Peabody Terrace 609
Cambridge, Mass. 02138

Dear Mr. Doyle:

Thank you for your letter with the example of a graph that is transitive on the vertices and edges but is not 1-transitive.

As you supposed there has been some work on the problem since my book was published, and such a graph is already known. It is described in a paper by Professor I.Z. Bouwer (of the University of New Brunswick, Fredericton, New Brunswick). The paper appears in the Canadian Mathematical Bulletin Vol. 13 (1970), pp 231-237. Bouwer mentions an earlier solution due to the late J. Folkman.

You might do well to consult Professor R.M. Foster (142 Mt. Hermon Way, Ocean Grove, N.J. 07756). He has a collection of symmetrical graphs, built up over some 40 years. He could probably say if yours was new.

Yours sincerely,

W T Tutte

WTT/js

W.T. Tutte

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142 Mount Hermon Way
Ocean Grove, New Jersey, 07756
May 5, 1976

Dear Mr. Doyle:

Your letter of April 25th was of great interest to me. So far as I am aware, that particular graph, with 27 vertices, has never appeared in the literature. But I must hasten to add that for the last half dozen years I have been immured in this godforsaken community, without access to decent library facilities (since I no longer drive). The only graphs known to me (transitive on the vertices and edges but not 1-transitive) are those discussed by Bouwer in his 1970 paper. I suggest that you write to him: Professor Izak Z. Bouwer, Department of Mathematics, University of New Brunswick, Fredericton, New Brunswick E3B 5A3, Canada.

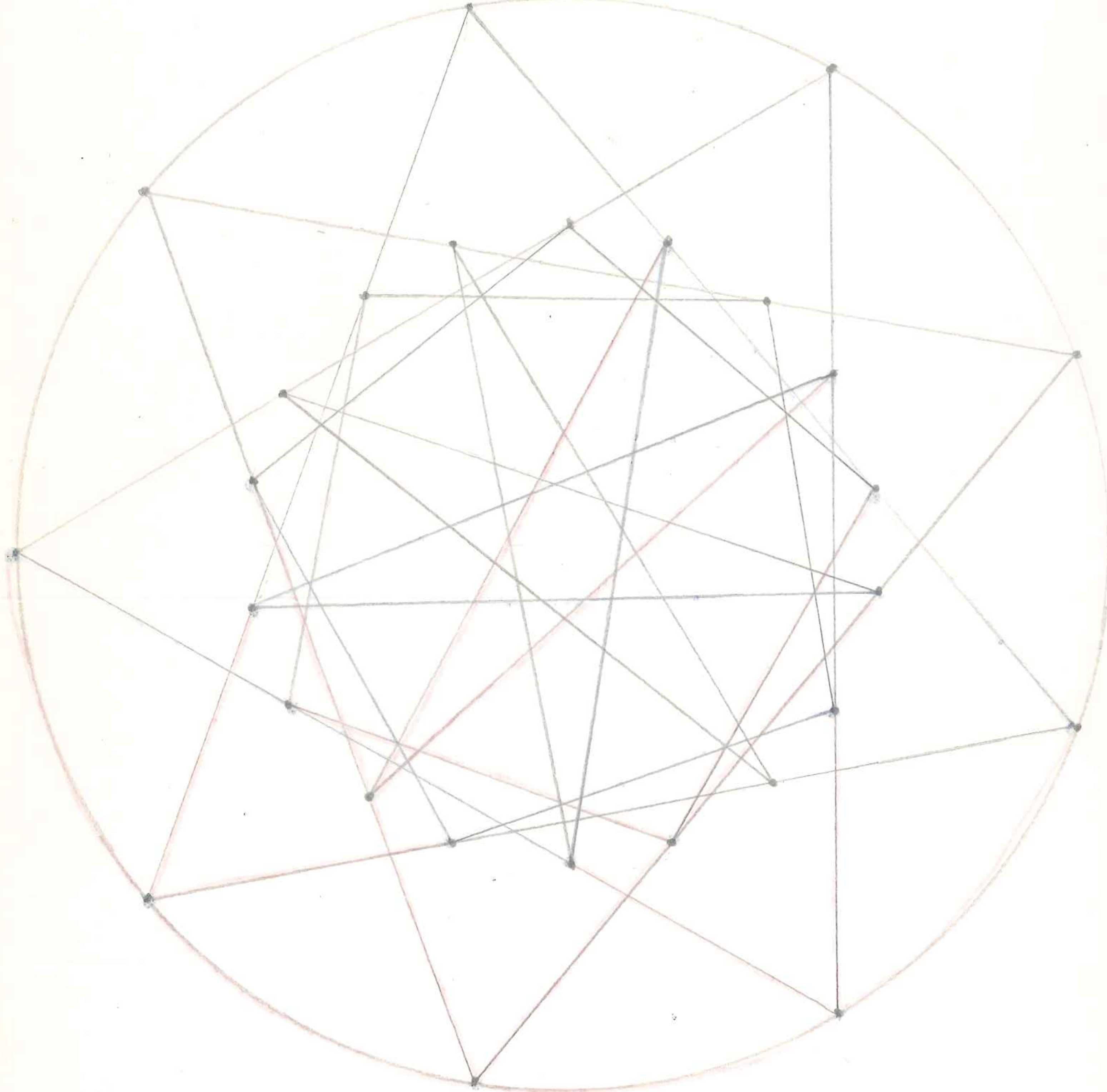
On the other hand, your graph is easily derived from Bouwer's graph X(2,6,9), of valency four with 54 vertices. These vertices occur in discrete pairs of vertices at a distance apart equal to the diameter of the graph. Each such pair of vertices can be identified with a vertex of your graph. I must confess that I had never before thought of doing this until the receipt of your letter a week or so ago. Nor have I investigated any possibility of a similar procedure for Bouwer's graphs with a value of N greater than 2.

If I can be of any further service to you in this connection, please feel free to call upon me.

Sincerely yours,

Ronald M. Foster

Ronald M. Foster



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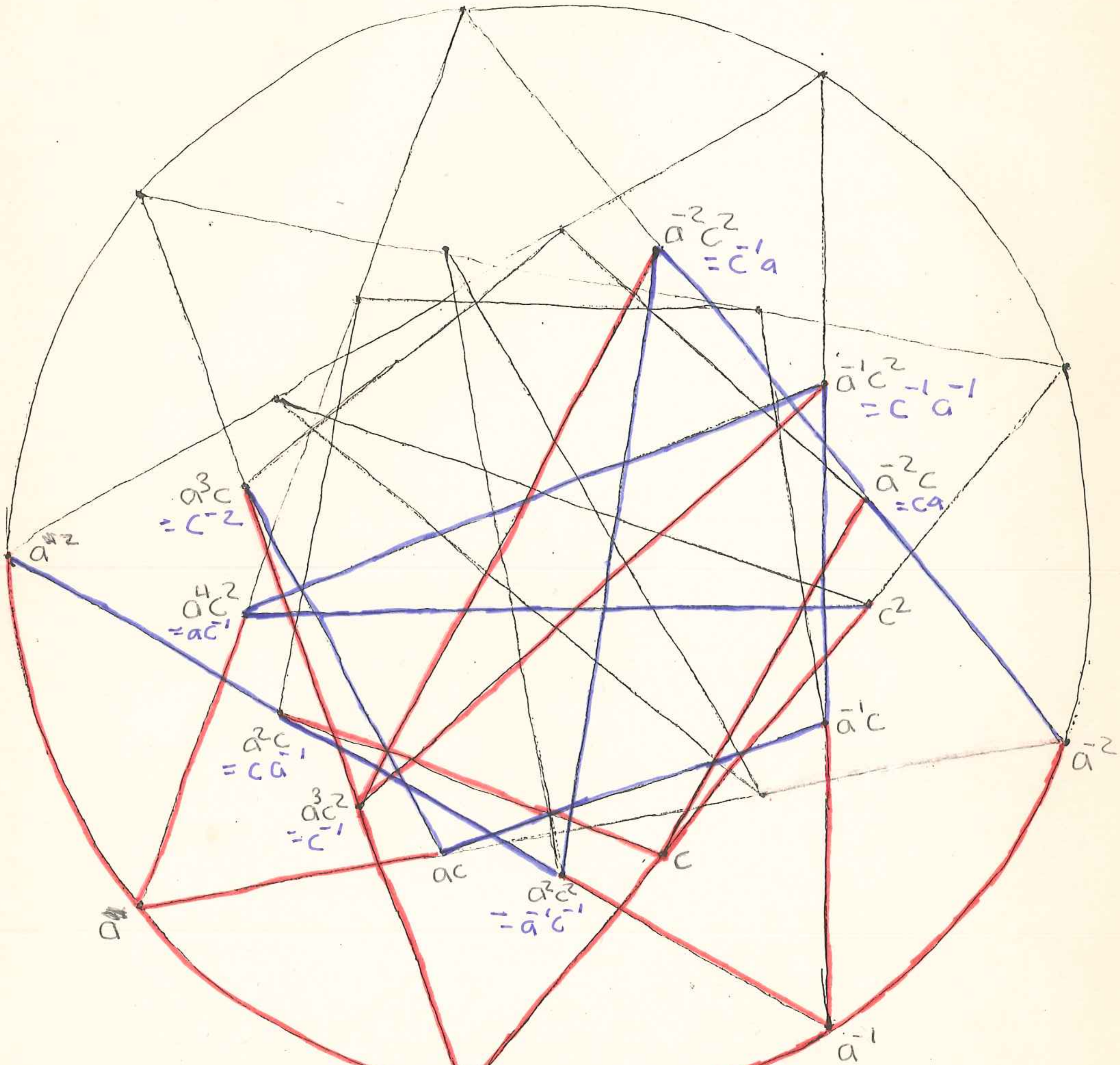
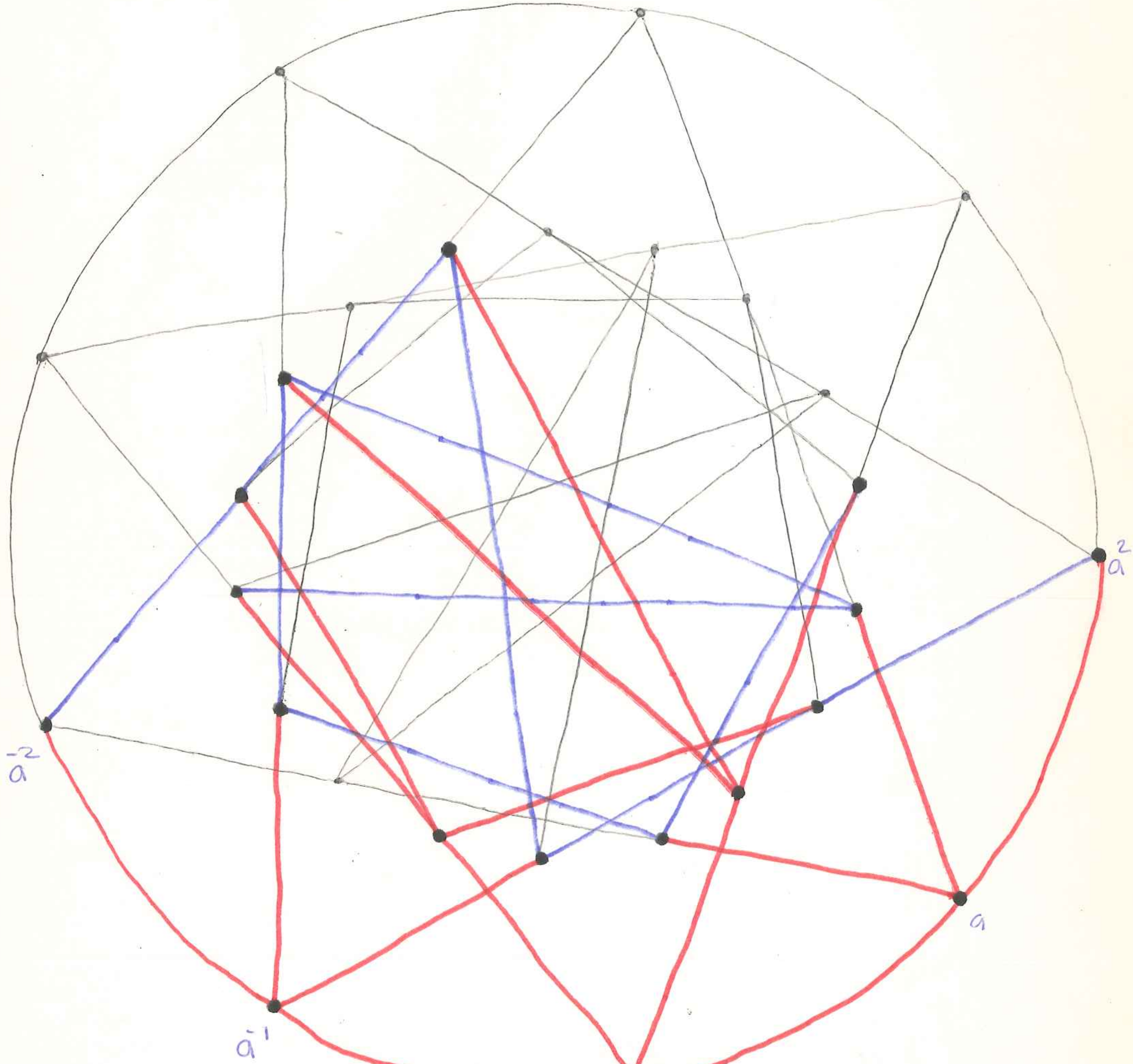
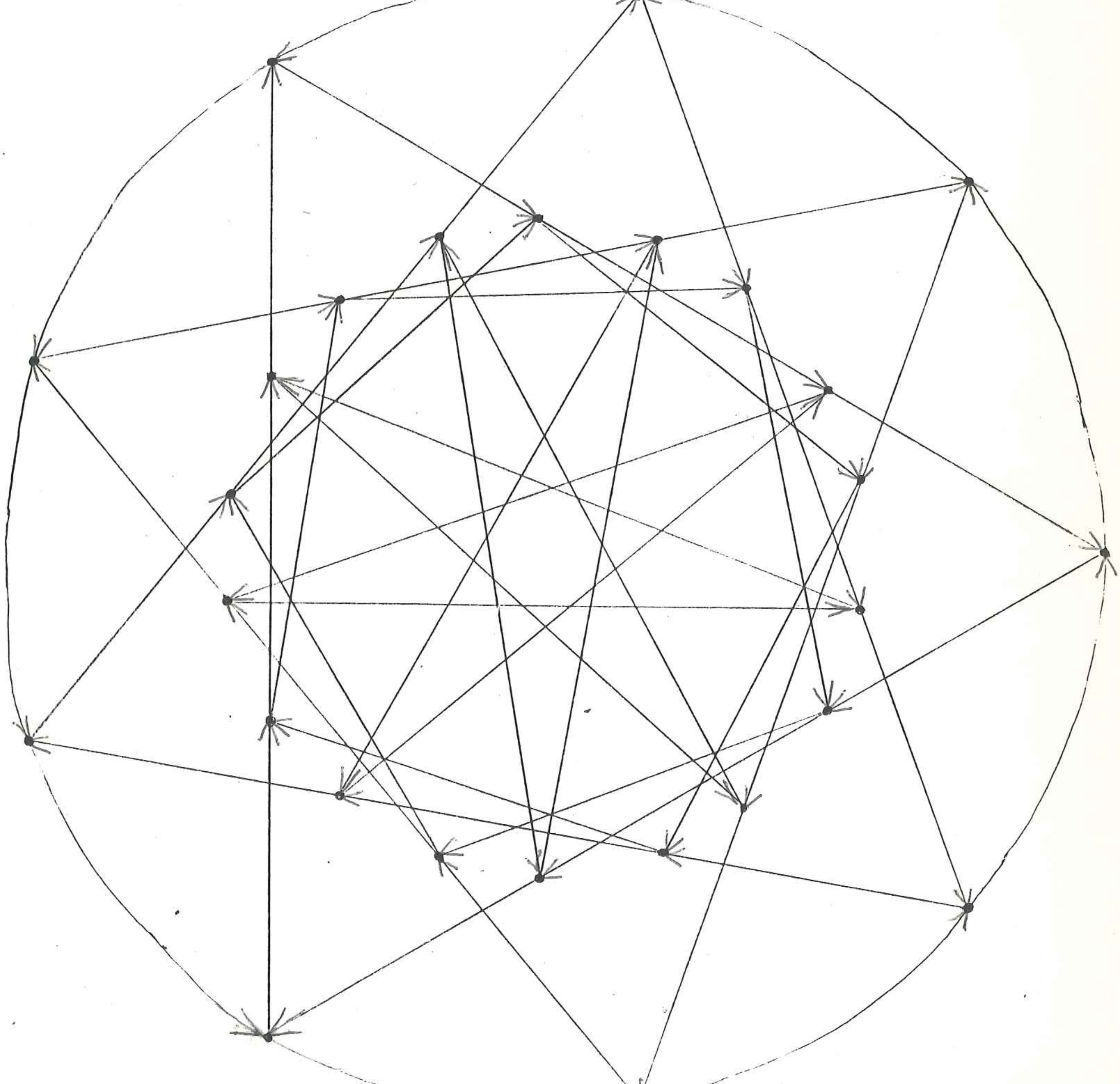

$a^{-2}c^2 = c^{-1}a$
$a^{-1}c^2 = c^{-1}a^{-1}$
$a^3c = c^{-2}$
$a^{-2}c = ca$
$a^4c^2 = ac^{-1}$
c^2
$a^{-1}c$
$a^2c^{-1} = ca^{-1}$
a^{-2}
$a^3c^2 = c^{-1}$
ac
c
$a^2c^2 = a^{-1}c^{-1}$
a
a^{-1}
1

(backwards)



a^2
a
1
a^{-1}
a^{-2}



LIST

GRAPH2   10 MAY 76   15:18

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50 DEF FNM(M,N)=M-INT(M/N)*N
100 LIBRARY "PLOTLIB***:TDI"
102 CALL "PSIZE":C(),10,7
110 DIM C(150),X(48,6),Y(48,6)
120 CALL "LIMITS":C(),-5/3,5/3,-3.5/3,3.5/3
190 LET P1=3.1415927
200 LET P=3
210 FOR I=0 TO P*P-1
220 LET X(I,0)=COS(2*P1*I/P/P)
230 LET Y(I,0)=SIN(2*P1*I/P/P)
235 CALL "LINE":C(),X(I,0),Y(I,0)
240 NEXTI
245 CALL "LINE":C(),1,0
250 FOR I=0 TO P*P-1
255 LET I1=FNM(I-P,P*P)
260 FOR J=1 TO P-1
270 LET X(I,J)=(1-J/P)*X(I,0)+J/P*X(I1,0)
280 LET Y(I,J)=(1-J/P)*Y(I,0)+J/P*Y(I1,0)
290 NEXTJ
300 NEXT I
310 FOR I=0 TO P-1
320 LET I1=FNM(I+P,P*P)
330 CALL "CONNECT":C(),X(I,0),Y(I,0),X(I1,0),Y(I1,0)
340 FOR J=2 TO P
350 LET I1=FNM(I+J*P,P*P)
360 CALL "LINE":C(),X(I1,0),Y(I1,0)
370 NEXT J
380 NEXT I
400 FOR J=1 TO P-1
405 CALL "LIFT":C()
410 LET A=0
430 CALL "LINE":C(),X(0,J),Y(0,J)
440 FOR K=1 TO P*P
450 LET A=FNM(A-J*P+1,P*P)
460 CALL "LINE":C(),X(A,J),Y(A,J)
470 NEXT K
480 NEXT J
485 CALL "FINISH":C()
487 INPUT Q$
488 IF Q$="N" THEN 610
490 LET S=.0075
500 FOR I=0 TO P*P-1
510 FOR J=0 TO P-1
520 LET X=X(I,J)
530 LET Y=Y(I,J)
540 CALL "CONNECT":C(),X-S,Y-S,X+S,Y-S
550 CALL "LINE":C(),X+S,Y+S
560 CALL "LINE":C(),X-S,Y+S
570 CALL "LINE":C(),X-S,Y-S
580 NEXT J
590 NEXT I
600 CALL "FINISH":C()
610 END
READY
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