Introduction to finite mathematics

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Preface to the second edition

THE FIRST EDITION of this book was published in January, 1957. Since that time we and our colleagues at various institutions have had experience teaching its material to a large number of students. We therefore felt it appropriate that a second edition should be prepared that would reflect the experience we have had.

The changes to the basic core (the unasterisked sections) of the book are minor, and involved the rewriting of a few sections and the addition of supplementary exercises. We have also added between ten to fifteen percent of new material, most of it in asterisked (optional) sections. A brief summary of the more important changes is as follows: the section on logical possibilities has been rewritten; a new section on counting techniques has been added; Bayes theorem replaces geometric probabilities; the section on the law of large numbers and central limit theorem has been split in two; the section on linear equations has been rewritten and a flow diagram added; the section on the inverse of a matrix has been rewritten; the material on Markov chains has been expanded; the material on linear programming has been completely rewritten; game theory has had minor rewriting; the section on equivalence classes in communication networks is new; and the section on computer simulation is new.

Our objectives in the present edition of the book are the same as for the first edition; namely, to present a course in elementary modern mathematics which can appear early in a student's career. The applications discussed in the course are primarily to the biological and social sciences and hence provide a point of view, other than that given by physics—the application stressed in calculus courses—concerning the possible uses of mathematics.

The only prerequisite for this book is the mathematical maturity obtained from two and a half or more years of high school mathematics.

Without being dogmatic about the way the book should be used, we make the following suggestions for possible courses to be taught from it:

1. *A basic mathematics course*, covering the unasterisked sections of Chapters I–V and supplemented by some of the asterisked sections from these Chapters as well as selected sections of Chapter VI.

2. *A mathematics course for behavioral scientists*, covering the unasterisked sections of Chapters I–V and selected sections from Chapters VI and VII.

In addition to these courses it is possible to derive a brief introduction to probability and its applications from Chapters I–IV and VII; and it is also possible to derive a brief introduction to matrix theory and its applications from Chapters II, V, VI, and VII. We have included a bibliography at the end of each chapter to guide those interested in further reading.

At the end of each section there are a number of exercises. We have tried to grade them in order of increasing difficulty, and to provide two of each kind, one with and the other without an answer printed in the text. There are also supplementary exercises at the end of some of the sections. Some of these exercises are easier and some are more difficult than the earlier ones. They were added to help provide variety.

We wish to thank our colleagues, in many different institutions, who sent us suggestions for revisions. In particular, we are grateful to Professor A. W. Tucker, who has shown a strong and continuing interest in our work. We would also like to thank F. Mosteller and H. Roberts, whose stimulating criticisms of our computer simulation section helped us to improve it. To Dartmouth College and Carnegie Institute of Technology we offer our appreciation for providing facilities which made the preparation of this book possible. And, finally, we thank the staff of Prentice-Hall for their careful attention to editorial details.

J.G.K., J.L.S., G.L.T.

Preface to the first edition

In the usual undergraduate mathematical curriculum, the courses which a student takes during his first two years are those leading up to the calculus and the calculus itself. A few years ago, the department of mathematics at Dartmouth College decided to introduce a different kind of freshman course, which students could elect along with these more traditional ones. The new course was to be designed to introduce a student to some concepts in modern mathematics early in his college career. While primarily a mathematics course, it was to include applications to the biological and social sciences and thus provide a point of view, other than that given by physics, concerning the possible uses of mathematics.

In planning the proposed course, we found that there was no textbook available to fulfill our needs and, therefore, we decided to write such a book. Our aim was to choose topics which are initially close to the students' experience, which are important in modern day mathematics, and which have interesting and important applications. To guide us in the latter we asked for the opinions of a number of behavioral scientists about the kinds of mathematics a future behavioral scientist might need. The main topics of the book were chosen from this list.

Our purpose in writing the book was to develop several topics from a central point of view. In order to accomplish this on an elementary level, we restricted ourselves to the consideration of *finite* problems, that is, problems which do not involve infinite sets, limiting processes, continuity, etc. By so doing it was possible to go further into the subject matter than would otherwise be possible, and we found that the basic ideas of finite mathematics were easier to state and theorems about them considerably easier to prove than their infinite counterparts.

The first five chapters form a natural unit. The discussion of the set of logical possibilities (in Chapter I) leads to the idea of the truth set associated with a statement which, in turn, gives a natural way of defining the probability that a statement is true (in Chapter IV). The correspondence that exists among logical operations (Chapter I), set operations (Chapter II), and probability operations (Chapter IV) becomes especially transparent in the finite case. A very useful pedagogical device, that of a "tree" (a special type of diagram) is used in these chapters and in the rest of the book to illustrate and clarify ideas. In particular, this allows an introduction to the theory of stochastic processes in an elementary manner. The Markov chains here introduced help to motivate vector and matrix theory, which is presented in Chapter V.

In Chapter VI the student is introduced to two recent branches of mathematics that have proved useful in applications, namely, linear programming and the theory of games. We are able to explain the basic ideas of both relatively quickly because of the mathematical preparation given in the earlier chapters.

In our concluding Chapter VII we discuss several significant applications of mathematics to the behavioral sciences. These were selected for their interest both to mathematicians and to behavioral scientists. One topic was chosen from each of five sciences: sociology, genetics, psychology, anthropology, and economics. A reader may find it more difficult to read parts of this chapter than the earlier chapters, but it was found necessary to make it so in order that nontrivial applications could be taken up and pursued far enough to see the contributions mathematics makes. In teaching a course from our book we would not expect that all of the topics from this chapter would be used. We hope, however, that Chapter VII will serve as reference and self-study material for ambitious students.

The Committee on the undergraduate program of the Mathematical Association of America was planning a new freshman mathematics program at the same time we were planning our book. They had already written Part I of *Universal Mathematics*, which is an introduction to analytic geometry and the calculus, and were making plans for Part II. When the chairman of that committee learned of the similarity of the plans for our book to those for Part II of *Universal Mathematics*, he invited one of us to join his committee. We believe that our book agrees with the spirit of their recommendations. We are grateful for their permission to use some of their illustrations, of which the applications to voting problems are the principal ones.

The report of the Committee on Mathematical Training of Social Scientists of the Social Science Research Council appeared after our plans were completed. We were pleased to note that on many questions we had reached the same conclusions as had that committee. They recommend two years of training, about half in the calculus and half along the lines here discussed. A semester course based on our book together with a semester of calculus would give the student a distribution in the proportions recommended by that committee.

The basic core of the book consists of the unasterisked sections of Chapters I-V. This material should be covered in every course. Flexibility is provided by the inclusion of additional material, the optional (asterisked) sections of these chapters, Chapter VI, and Chapter VII. By emphasizing the first five chapters, the course would be a basic mathematics course. By aiming at Chapter VII and taking up several of these applications, the course can be designed as a mathematics course suited for the behavioral scientist. Chapter VI is appropriate as supplementary material for either type of course. We have included a bibliography at the end of each chapter to guide those interested in further reading.

The only prerequisite for this book is the mathematical maturity obtained from two and a half or more years of high school mathematics. Our book has been tried successfully in a freshman course at Dartmouth College and for supplementary reading in other courses. It has also been used in a mathematics course for faculty members in the behavioral sciences.

We wish to thank Dartmouth College for releasing us from part of our teaching duties to enable us to prepare this book. Thanks are also due to A. W. Tucker for his valuable advice and to our colleagues in the mathematics department at Dartmouth for their many helpful

suggestions. We are also grateful to James K. Schiller for reading the manuscript and for providing the reactions of a student. Finally we wish to thank Joan Snell, Margaret P. Andrews, and Stephen Russell for their invaluable aid in the preparation of the manuscript.

J.G.K., J.L.S., G.L.T.

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