

CHRISTIANI HUGENII

LIBELLUS  
DE  
RATIOCINIIS  
IN  
*LUDO ALEAE*

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OR,  
The VALUE of all  
CHANCES  
IN  
Games of Fortune;  
CARDS, DICE, WAGERS,  
LOTTERIES, &c.  
Mathematically Demonstrated.

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[This document consists of a reprint of an English translation of the first book on probability theory ever published, viz. Christiaan Huygens' *De Ratiociniis in Ludo Aleae* which was published in Latin in 1657. The reprint of the translation is complete, except that the dedication to Dr. Richard Mead by W. Browne has been omitted. The original pagination is indicated in square brackets.]

[1]

### The Value of Chances

ALTHOUGH in games depending entirely upon Fortune, the Success is always uncertain; yet it may be exactly determin'd at the same time, how much more likely one is to win than lose. As, if any one shou'd lay that he wou'd throw the Number *Six* with a single die the first throw, it is indeed uncertain whether he will win or lose; but how much more probability there is that he shou'd lose than win, is easily determin'd, and easily calculated. So likewise, if I agree with another to play the first Three Games for a certain Stake, and I have won one of my Three, it is yet uncertain which of us shall first get his third Game; but the Value of my Expectation and his likewise may be exactly discover'd; and consequently it may be determin'd, if we shou'd both agree to give over play, and leave the remaining Games unfinish'd, how much more of the Stake comes to my Share than his; [2] or, if another desired to purchase my Place and Chance, how much I might just sell it for. And from hence an infinite Number of Questions may arise between two, three, four, or more Gamesters: The satisfying of which being a thing neither vulgar nor useless, I shall here demonstrate in few words, the Method of doing it; and then likewise explain particularly the Chances that belong more properly to Dice.

### POSTULAT

AS a Foundation to the following *Proposition*, I shall take Leave to lay down this Self-evident Truth: *That any one Chance or Expectation to win any thing is worth just such a Sum, as wou'd procure in the same Chance and Expectation at a fair Lay.* As for Example, if any one shou'd put 3 Shillings in one Hand, without telling me know which, and 7 in the other, and give me Choice of either of them; I say, it is the same thing as if he shou'd give me 5 Shillings; because with 5 Shillings I can, at a fair Lay, procure the same even Chance or Expectation to win 3 or 7 Shillings.

### PROP. I.

*If I expect  $a$  or  $b$ , and have an equal chance of gaining either of them, my Expectation is worth  $\frac{a+b}{2}$ .*

[3] To trace this Rule from its first Foundation, as well as demonstrate it, having put  $x$  for the value of my Expectation, I must with  $x$  be able to procure the same Expectation at a fair Lay. Suppose then that I play with another upon this Condition, That each shall stake  $x$ , and he that wins give the Loser  $a$ . 'Tis plain, the Play is fair, and that I have upon this Agreement an even Chance

to gain  $a$ , if I lose the Game; or  $2x - a$ , if I win it: for I then have the whole stake  $2x$ , out of which I am to pay my Adversary  $a$ . And if  $2x - a$  be supposed equal to  $b$ , then I have an even Chance to gain either  $a$  or  $b$ . Therefore putting  $2x - a = b$ , we have  $x = \frac{a+b}{2}$ , for the Value of my Expectation. *Q.E.I.*

THE Demonstration of which is very easy: For having  $\frac{a+b}{2}$ , I can play with another, who shall likewise stake  $\frac{a+b}{2}$ , upon Condition that the Winner shall pay the Loser  $a$ . By which means I must necessarily have an equal Expectation to gain  $a$ , if I am Loser, or  $b$ , if I am Winner; for then I win  $\frac{a+b}{2}$ , the Whole Stake, out of which I am to pay the Loser  $a$ . *Q.E.D.*

IN Numbers. If I have an equal Chance to 3 or 7, then my Expectation is, by this *Proposition*, worth 5, and it is certain I can with 5, again procure the same Expectation: For if Two of us stake 5 a piece upon this Condition, That he that wins pay the other 3, 'tis plain the Lay is just [4] and that I have an even Chance to come off with 3, if I lose, or 7 if I win; for then I gain 10, and pay my Adversary 3 out of it. *Q.E.D.*

#### PROP. II.

*If I expect  $a$ ,  $b$ , or  $c$ , and each of them be equally likely to fall to my Share, my Expectation is worth  $\frac{a+b+c}{3}$ .*

TO calculate which, I again put  $x$  for the value of my Expectation: Therefore having  $x$ , I must be able, by fair Gaming, to procure the same Expectation. Supposing then I play with two others upon this Condition, That every one of us stake  $x$ ; and I agree with one of them, that which soever of us Two wins, shall give the loser  $b$ ; and with the other, that which soever of us Two wins, shall give the Loser  $c$ . It appears evidently, that the Lay is very fair, and that I have by this means an equal Chance to gain  $b$ , if the first wins; or  $c$  if the second wins; or  $3x - b - c$ , if I win my self; for then I have the whole Stake  $3x$ , but of which I give  $b$  to one, and  $c$  to the other. But if  $3x - b - c = a$ , we shall find  $x = \frac{a+b+c}{3}$ , for the value of my Expectation. *Q.E.I.*

AFTER the same Manner, an even Chance to  $a$ ,  $b$ ,  $c$  or  $d$ , will be found worth  $\frac{a+b+c+d}{4}$ . And so on.

[5]

#### PROP. III.

*If the number of Chances I have to gain  $a$ , be  $p$ , and the number of Chances I have to gain  $b$ , be  $q$ . Supposing the Chances equal; my Expectation will then be worth  $\frac{ap+bq}{p+q}$ .*

TO investigate this Rule, I again put  $x$  for the value of my Expectation which must consequently procure me the same Expectation in fair Gaming. I take therefore such a Number of Gamesters as may, including my self, be equal to  $p + q$ , every one of which stakes  $x$ ; so that the whole stake is  $px + qx$ , and all play with an equal Expectation of winning. With so many Gamesters as are expressed by the Number  $q$ , I agree singly, that whoever of them wins, shall give me  $b$ ; and if I win, he shall have  $b$  of me: And with the rest, express'd by  $p - 1$ ,

I singly make this Agreement, That whoever of them wins, shall give me  $a$ ; and if I win, he shall receive  $a$  of me. It is evident, our playing upon this Condition is fair, no Body having any injury done him; and that my Expectation of  $b$  is  $q$ ; my Expectation of  $a$  is  $p - 1$ ; and my Expectation of  $px + qx - bq - ap + a$  (*i.e.* of winning) is 1: for then I gain the whole Stake  $px + qx$ , out of which I must pay  $b$ , to every one of the Gamesters  $q$ , and  $a$  to every one of the Gamesters  $p - 1$ , which together makes  $bq + ap - a$ . If therefore  $px + qx - bq - ap + a$  be equal to  $a$ , I shou'd have  $p$  Expectations of  $a$  (for I had  $p - 1$  Expectations of  $a$ , and [6] 1 Expectation of  $px + qx - bq - ap - a$ , which is now supposed equal to  $a$ .) and  $q$  Expectations of  $b$ ; and consequently am again come to my first Expectation. Therefore  $px + qx - bq - ap + a = a$ , and consequently  $x = ap + bq$ , is the value of my Expectation. *Q.E.I.*

IN Numbers. If I have 3 Expectations of 13 and 2 Expectations of 8, the value of my Expectations wou'd by this Rule be 11. And it is easy to show, that having 11, I cou'd again come to the same Expectations. For playing against Four more, and every one of us staking 11; with Two of them I agree singly, that he that wins shall give me 8; or to give him 8, if I win: And with the other Two in like manner, that which soever wins, shall give me 13; or give him so much if I win. The Play is manifestly fair, and I have just 2 Expectations of 8, if either of the Two that promis'd me 8 shou'd win; and 3 Expectations of 13, if either of the Two that are to pay me 13 shou'd win, or if I win my self, for then I gain the whole Stake, which is 55; from which, deducting 13, a piece for the last Two I bargain'd with, and 8 a piece for the other Two, there remain 13 for my self. *Q.E.D.*

[7]

#### PROP. IV.

*To come to the Question first propos'd, How to make a fair Distribution of the Stake among the several Gamesters, whose Chances are unequal? The best way will be to begin with the most easy Cases of that Kind.*

SUPPOSING therefore that I play with another upon this Condition, That he who gets the first three Games shall have the Stake; and that I have won two of the three, and he only one. I desire to know, if we agree to leave off and divide the Stake, how much falls to my Share.

IN the first place we must consider the number of Games still wanting to either Party: For it is plain, that supposing we had agreed the Stake shou'd be deliver'd to him that shou'd win the first twenty Games, and I had won nineteen of them, and the other only eighteen; my Chances wou'd have then been just so much better than his, as it is in the present Case, where I am supposed to have won two out of the three, and he only one: because in both Cases there remains but one Game for me to win and two for him.

MOREOVER to find how to share the Stake, we must have regard to what would happen, if both play'd on: For it is manifest, that if I win the next Game, my Number is compleated, and the Stake, which call  $a$ , is mine. But if the other gets the next Game, then both our Chances will be even, because we want but

one Game apiece, and each of them [8] worth  $\frac{1}{2}a$ . But it is plain, I have an equal Chance to win or lose the next Game; and consequently an equal Chance to gain  $a$ , or  $\frac{1}{2}a$ ; which by *Prop.* 1. is worth half the Sum of them both, *i.e.*  $\frac{3}{4}a$ . *Q.E.I.*

MY Playfellow's Share, which of course must be the remaining  $\frac{1}{4}a$ , might be first found after first found after the same manner. From whence it appears, That he who would play in my room, ought to give me  $\frac{3}{4}a$  for my Chance; and consequently that whoever undertakes to win one Game, before another shall win two, may lay 3 to 1 Odds.

PROP. V.

*Suppose I want one Game of being up, and my Adversary wants three; How must the Stakes be divided?*

LET us again consider what wou'd be the Consequence, if I shou'd get the next Game; 'tis plain I should win the Stake, suppose  $a$ ; but if the other shou'd get it, he wou'd still want two Games and I but one; and consequently our Case would be the same with that mention'd in the foregoing Proposition and my Share,  $\frac{3}{4}a$ , as is there demonstrated. [9] Therefore, since I have an equal Chance to gain  $a$ , or  $\frac{3}{4}a$ , my Expectation must by *Prop.* 1. be worth  $\frac{7}{8}a$ ; and my Adversary's Share, the remaining  $\frac{1}{8}a$ . So that my Chance is to his, as 7 to 1. *Q.E.I.*

AND as the *Solution* of the foregoing Case is necessary to solving this last, so is the *Solution* of this last necessary to solving the following one, where I am supposed to want but one Game, and my Adversary four; for then my Share will be found, after the same manner, to be  $\frac{15}{16}$  of the Stake, and his,  $\frac{1}{16}$ .

PROP. VI.

*Suppose I have two Games to get, and my Adversary three.*

THEREFORE after the next Game, I shall want but one more, and he three (in which Case my Share, by the foregoing *Prop.* is  $\frac{7}{8}a$ ) or we shall want two a piece, and then my Share is  $\frac{1}{8}a$ , both our Chances being equal. But I have an even Chance to win or lose the next Game, and consequently have an equal Expectation of obtaining  $\frac{7}{8}a$ , or  $\frac{1}{2}a$ , w<sup>c</sup> [10] by *Prop.* 1 is worth  $\frac{11}{16}a$ . So that eleven Parts of the Stake fall to my Share, and five to his. *Q.E.I.*

PROP. VII.

*Suppose I want two Games, and my Adversary four.*

THEREFORE it will either fall out, that by winning the next Game, I shall want but one more, and he four, or by losing it I shall want two, and he shall want three. So that by *Schol. Prop.* 5. and *Prop.* 6., I shall have an equal Chance for  $\frac{15}{16}a$  or  $\frac{11}{16}a$ , which, by *Prop.* 1 is just worth  $\frac{13}{16}a$ . *Q.E.I.*

COROLL.

FROM whence it appears, that he that is to get two Games, before another shall get four, has a better Chance than he is to get one, before another gets two Games. For in this last Case, namely of 1 to 2 his Share by Prop. 4 is but  $\frac{3}{4}a$ , which is less than  $\frac{13}{16}a$

PROP. VIII.

*Suppose now there are three Gamesters and that the first and second want a game a piece, and the third wants two Games.*

[11] TO find therefore the Share of the first Gamesters, we must again examine what he wou'd gain, if either he himself, or one of the other two gets the next Games: If he gets it, he wins the whole Stake  $a$ ; if the second gets it, because he likewise wanted but one Game, he has the Stake, and the first gets 0; and if the third gets it, then they all three want one Game a piece, and the Share of each of them will consequently be  $\frac{1}{3}a$ . The first Gamester therefore has an equal Expectation of gaining  $a$ , or 0, or  $\frac{1}{3}a$ , (since each has the same likelihood of winning the next Game,) which, by Prop. 2. is worth  $\frac{4}{9}a$ . The Share of the second will be likewise  $\frac{4}{9}a$ , and there will be  $\frac{1}{9}a$  remaining for the third; whose Share might, after the same manner, be found separately from the others, and theirs determin'd by that. *Q.E.I.*

PROP. IX.

*To find the several Shares of as many Gamesters, as we please, some of which shall want more Games, others fewer; we must consider what he, whose Share we want to find, wou'd gain, if he, or any one of the others wins the next Game: Then adding together what he wou'd gain in all those particular Cases, and dividing the sum by the Number of Gamesters, the Quotient gives the particular Share required.*

[12] SUPPOSE, for Example, there were three Gamesters  $A$ ,  $B$ , and  $C$ , and  $A$  wanted one Game of being up, and  $B$  and  $C$  wanted two apiece, and I desire to find the Share that  $B$  has in the Stake  $q$ .

FIRST of all we must see what will happen to  $B$ , if either he, or  $A$ , or  $C$  wins the following Game.

IF  $A$  wins it, he is up and consequently  $B$  gets 0. If  $B$  himself wins it, he and  $A$  will still want a Game apiece, and  $C$  will want two; and consequently  $B$ , by Prop. 8., gets  $\frac{4}{9}q$ . But lastly, if  $C$  wins it, then  $A$  and  $C$  will still want two; and consequently  $B$ , by Prop. 8. will in this Case get  $\frac{1}{9}q$ .

NOW the three several Gains of  $B$  in all these particular Cases, which are 0,  $\frac{4}{9}q$ , and  $\frac{1}{9}q$ , added together make  $\frac{5}{9}q$ ; which Sum being divided by 3, the number of Gamesters, gives  $\frac{5}{27}q$  for the Share of  $B$ . *Q.E.I.*

THE Demonstration of this is plain from Prop. 2. For since  $B$  has an equal Chance to 0,  $\frac{4}{9}q$  or  $\frac{1}{9}q$ , his Expectation is, by that Prop. worth  $\frac{0+\frac{4}{9}q+\frac{1}{9}q}{3}$  i.e.  $\frac{5}{27}q$ . And 'tis evident, this Divisor 3, is the Number of Gamesters. *Q.E.D.*

[13] BUT in order to find what any one will gain in every particular Case, supposing himself or any of the rest should win the next Game; the more simple

and intermediate Cases must first be investigated. For as without that calculated in *Prop.* 8. where the Games wanting were 1,1,2; so likewise every single Person's Share, in case the Games wanting were 1,2,3, cannot be found, without first determining the Case where the Games wanting are 1,2,2 (which is already done in *Prop.* 9. and that likewise where the Games wanting are 1,1,3, which may, by *Prop.* 8. be easily calculated. And after this manner may be resolv'd all the Cases comprehended in the following Table, and others, ad Infinitum.

TABLE

[14]

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TABLE for Three Gamesters

<i>Games wanting Shares.</i>	$\frac{1. 1. 2.}{4. 4. 1.}$ 9.	$\frac{1. 2. 2.}{17. 5. 5.}$ 27.	$\frac{1. 1. 3.}{13. 13. 1.}$ 27.	$\frac{1. 2. 3.}{19. 6. 2.}$ 27.
<i>Games wanting Shares.</i>	$\frac{1. 1. 4.}{40. 40. 1.}$ 81.	$\frac{1. 1. 5.}{121. 121. 1.}$ 243.	$\frac{1. 2. 4.}{178. 58. 7.}$ 243.	$\frac{1. 2. 5.}{542. 179. 8.}$ 729.
<i>Games wanting Shares.</i>	$\frac{1. 3. 3.}{65. 8. 8.}$ 81.	$\frac{1. 3. 4.}{616. 82. 31.}$ 729.	$\frac{1. 3. 5.}{629. 87. 13.}$ 729.	
<i>Games wanting Shares.</i>	$\frac{2. 2. 3.}{34. 34. 13.}$ 81.	$\frac{2. 2. 4.}{338. 338. 53.}$ 729.	$\frac{2. 2. 5.}{353. 353. 23.}$ 729.	$\frac{2. 3. 3.}{133. 55. 55.}$ 243.
[Continuation of same line in original]			$\frac{2. 3. 4.}{451. 195. 83.}$ 729.	$\frac{2. 3. 5.}{1433. 635. 119.}$ 2187.



[15] AS to what belongs to DICE, the Questions propos'd concerning them are, In how many Times we may venture to throw Six, or any other Number, with a single Die, or two Sixes with two Dice, or three Sixes with three Dice; and such-like.

TO resolve which, we must observe, First, That there are six several Throws upon one Die, which all have an equal probability of coming up. That upon two Dice there are 36 several Throws, equally liable to be thrown, for any one of the six Throws of one Die may come up with every one of the six Throws of the other; and so 6 times 6 will make 36 Throws. So likewise, that three Dice have 216 several Throws; for the 36 Throws on the two Dice may happen together with any one of the 6 Throws of the third Die; and so 6 times 36 will make plain 216 Throws; for the 36 Throws on the two Dice may happen together with any one of the 6 Throws of the third Die; and so 6 times 36 will make 216 Throws. After the same manner, 'tis plain, fair dice will have 6 times 216, *i.e.* 1296 Throws; and so on, may we calculate the Throws on any Number of Dice, taking always at the addition of every Die 6 times the preceding Number of Throws.

IT is farther to be observ'd, that upon two Dice there is only one Throw that can produce 2 or 12, and two Throws that can produce 3 or 11. For if we call the Dice *A* and *B*, 'tis plain, the throw 3 may be made up of the 1 of *A* and 2 of *B*; or of the 1 of *B* and 2 of *A*. So likewise will 11 be produced by the 5 of *A*, and 6 of *B*; or by the 5 of *B*, and 6 of *A*. The Number 4 may be thrown three Ways, by 1 of *A* and 3 of *B*; or 3 of *A* and 1 of *B*; or 2 of *A* and 2 of *B*.

[16] X may likewise be thrown three several Ways.

V or IX has 4 several Throws.

VI or VIII has 5 Throws.

VII has 6 Throws.

Upon 3 Dice, the Numbers	$\left( \begin{array}{c} 3 \text{ or } 18 \\ 4 \text{ or } 17 \\ 5 \text{ or } 16 \\ 6 \text{ or } 15 \\ 7 \text{ or } 14 \\ 8 \text{ or } 13 \\ 9 \text{ or } 12 \\ 10 \text{ or } 11 \end{array} \right)$	may come up by *	$\left( \begin{array}{c} 1 \\ 3 \\ 6 \\ 10 \\ 15 \\ 21 \\ 25 \\ 27 \end{array} \right)$	several Throws *
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[\* These words printed vertically in the original]

#### PROP. X.

*To find how many Throws one may undertake to throw the Number 6 with a single Die.*

IF any one wou'd venture to throw *Six* the first Throw, 'tis plain there is but 1 Chance by which he might win the Stake; and 5 Chances by which he might

lose it: For there are 5 Throws against him and only 1 for him. Let the stake be called  $a$ . Since therefore he has one Expectation of  $a$ , and 5 Expectations of 0, his Chance is, by *Prop.* 2. worth  $\frac{1}{6}a$ ; and consequently there remains to his Adversary  $\frac{5}{6}a$ . So that he that wou'd undertake to throw *Six* the first Throws, must lay only 1 to 5.

[17] HE that wou'd venture to throw *Six* once in two Throws, may calculate his Chance after the following manner: If he throws *Six* the first Throw, he gains  $a$ ; if the contrary happens, he has still another Throw remaining, which by the foregoing Case, is worth  $\frac{1}{6}a$ , the Value of which, by *Prop.* 3. is  $\frac{11}{36}a$ . And consequently there remains  $\frac{25}{36}a$ , to the other that lays with him. So that their several Chances, or the Values of their several Expectations, bear the Proportion of 11 to 25, *i.e.* less than 1 to 2.

HENCE, after the same Method, the Chance of him who wou'd venture to throw *Six* one in three Throws, may be investigated and found worth  $\frac{91}{216}a$ , so that he may lay 91 against 125; which is a little less than 3 to 4.

HE who undertakes to throw it once in four Times, has a Chance worth  $\frac{671}{1296}a$ , and may lay 671 to 625, *i.e.* something more than 1 to 1.

HE who undertakes to throw it once in five Times, has a Chance worth  $\frac{4651}{7776}a$ , and may lay 4651 against 3125, *i.e.* something less than 3 to 2.

[18] HE who undertakes the same in 6 Throws has a Chance worth  $\frac{31031}{46656}a$ , and may lay 31031 to 15625, *i.e.* a little less than 2 to 1.

AND thus the Problem be resolv'd in what Number of Throws we please.  
*Q.E.I.*

BUT it is possible for us to proceed after a more compendious Method, as shall be shown in the following Proposition, without which the Calculation wou'd otherwise be much more prolix.

#### PROP. XI.

*To find in how many Throws one may venture to throw the Number 12 with two Dice.*

IF any one shou'd pretend to throw 12 the first throw, 'tis plain he has but one Chance of winning, *i.e.* of gaining  $a$ ; and 35 Chances of losing, or gaining 0, because there are in all, 36 several Throws. And consequently his Expectation, by *Prop.* 3. is worth  $\frac{1}{36}a$ .

HE that undertakes to do it in two Throws, if it comes up in the first Throw, will obtain  $a$ ; and if it does not he has yet one Throw to come, which, by what has been said before, is worth  $\frac{1}{36}a$ . But there is only one Chance for throwing 12 the first Throw and 35 Chances against it: Therefore since he has 1 Expectation of  $a$  and 35 of  $\frac{1}{36}a$ , his [19] Chance by *Prop.* 3. is worth  $\frac{71}{1296}a$ ; and his Adversary's the remaining  $\frac{1225}{1296}a$

FROM these Two Cases we can determine the value of his Chance who ventures to do the same in Four Throws, without considering the Chance of him that undertakes to do it in Three.

FOR he that ventures to throw 12 once in four times throwing, if he does not throw it the first or second time, gains  $a$ ; and if the contrary happens, he has

still two Throws more, which, by what has been said before, are worth  $\frac{71}{1296}a$ : For which Reason likewise, he must have 71 Chances for throwing 12 in one of the two first, and 1225 Chances against it. Therefore at his beginning to throw he has 71 Expectations of a, which, by *Prop. 2.*, are worth  $\frac{178991}{1679616}a$ : And the value of his Chance that plays against him will be the remaining  $\frac{1500625}{1679616}a$ . Which shows that their Chances are to one another, as 178991 to 1500625.

FROM which likewise, without calculating any other Cases for that Purpose, may be found by the same Way of Reasoning, the worth of his Expectation who undertakes to throw two *Sixes* once in 8 Throws. And from thence the worth of his Expectation [20] who ventures to do the same in 16 Throws. And from the Expectation of this last being found, together with his also who ventures it in 8 Throws, may be determined the value of his Expectation who undertakes it in 24 Throws. In which Operation, because the principal Question is, In what Number of Throws one may lay an even Wager to throw two *Sixes*; we may cut off some of the hindermost Figures from the long Numbers that arise in the midst of the Calculations, and which wou'd otherwise encrease prodigiously. And by this Means, I find he that undertakes it in 24 Throws wants something of an even Chance to win; and he that lays to do it in 25 Throws, has something the better of the Wager.

#### PROP. XII.

*To find with how many Dice one may undertake to throw two Sixes the first Throw.*

THIS is the same thing as if we wou'd know, in how many Throws one may undertake to throw two *Sixes* with a single Die. He that undertakes it in three Throws, if he does not happen to throw one *Six* the first Throw, has yet two more to come, which as before, are worth  $\frac{1}{36}a$ . But if the first Throw chance to be a *Six*, he has two Throws more to throw one *Six*, which by *Prop. 10.* are worth  $\frac{11}{36}a$ . [21] Now 'tis plain that there is one Chance for throwing a *Six* the first Throw, and five Chances to the contrary: So that before he throws, he has one Chance for  $\frac{11}{36}a$ , 5 Chances of  $\frac{1}{36}a$ , which by *Prop. 3.*, are worth  $\frac{16}{216}a$ , or  $\frac{2}{27}a$ . After this manner, by continually taking one more throw, we find that we may in 10 Throws with one Die, or in one Throw with 10 Dice, undertake to throw two *Sixes*, and that with Advantage.

#### PROP. XIII.

*Supposing I lay with another to take one Throw with a pair of Dice upon these Terms, That if the Number 7 comes up, I shall win, and if 10 comes up, he shall win; and after this Bargain made, we consent to draw Stakes by a fair Division, according to the Value of our Chances in the present Contract: To find what shall be our several Shares.*

BECAUSE, of the 36 Throws upon two Dice, there are but 6 which consist of the Number 7 a piece, and 3 which consist of the Number 10 a piece; there remain 27 Throws, which, if any one of them chance to come up, will make us

neither win nor lose, and consequently entitle each of us to  $\frac{1}{2}a$ . But if none of those Throws should happen, I have 6 Chances for winning  $a$ , or and Chances of losing; or having 0, which by *Prop.* 3. is as good as if [22] I had  $\frac{2}{3}a$ . Therefore I have from the beginning 27 Chances for  $\frac{1}{2}a$  and 9 Chances for  $\frac{2}{3}a$ , which by *Prop.* 3. is worth  $\frac{13}{24}a$ ; and there remains to him that plays against me  $\frac{11}{24}a$ .

PROP. XIV.

*If my self and another play by turns with a pair of Dice upon these Terms, That I shall win if I throw the Number 7, or he if he throw 6 soonest, and he to have the Advantage of first Throw: To find the Proportion of our Chances.*

SUPPOSE my Chance worth  $x$ , and call the Stake  $a$ ; therefore his Chance will be  $= a - x$ . 'Tis plain then, that whenever it comes to his Turn to throw, my Chance ought to be  $= x$ . But when it is my Turn to throw, my Chance must be worth something more. Let its Value, then, be express'd by  $y$ . Now because of the 36 Throws upon a pair of Dice, 5 are made up of the Number 6 apiece, and may make my Adversary win, and 31 of them are against him, *i.e.* promote my Turn of throwing; I have, before he begins to throw, 5 Chances of obtaining 0, and 31 Chances of obtaining  $y$ , which, by *Prop.* 3. are worth  $\frac{31}{36}y$ . But my Chance from the beginning was supposed worth  $x$ , and therefore  $\frac{31}{36}y = x$ , and consequently [23]  $y = \frac{36}{31}x$ . It was further supposed that in my Turns of throwing, my Chance was worth  $y$ . But when I'm to throw, I have 6 Chances of gaining  $a$ , because there are 6 Throws of the Number 7 apiece, which wou'd give me the Game; and 30 Chances, which will bring it to my Adversary's Turn to throw, *i.e.* make me gain  $x$ ; which, by *Prop.* 3. are worth  $\frac{6a+30x}{36}$ . And because this  $= y$ , which was before found  $= \frac{36}{31}x$ ; therefore  $\frac{6a+30x}{36} = \frac{36}{31}x$ . From which Equation will be had  $x = \frac{31}{61}x$ , the Value of my Chance. And by consequence my Adversary's will be worth  $\frac{30}{61}x$ , so that the Proportion of my Chance to his is, as 31 to 30.

*For a concluding Ornament to this Work, we shall subjoin the following Problems.*

PROBLEM I.

$A$  and  $B$  play together with a pair of Dice upon this Condition, That  $A$  shall win if he throws 6, and  $B$  if he throws 7; and  $A$  is to take one Throw first, and then  $B$  two Throws together, then  $A$  to take two Throws together, and so on both of them the same, till one wins. The Question is, *What Proportion their Chances bear to one another?* Anfw. As 10355 to 12276.

[24]

PROBLEM II.

THREE Gamesters,  $A$ ,  $B$ , and  $C$ , taking 12 Counters, 4 of which are White, and 8 black, play upon these Terms: That the first of them that shall blindfold choose a white Counter shall win; and  $A$  shall have the first Choice,  $B$  the second, and  $C$  the third; and then  $A$  to begin again, and so on in their turns. *What is the Proportion of their Chances?*

PROBLEM III.

*A* lays with *B*, that out of 40 cards, *i.e.* 10 of each different Sort, he will draw 4, so as to have one of every Sort. And the Proportion of his Chance to that of *B*, is found to be as 1000 to 8139.

PROBLEM IV.

HAVING chosen 12 Counters as before, 8 black and 4 white, *A* lays with *B* that he will blindfold take 7 out of them, among which there shall be 3 black ones. *Quaere*, What is the Proportion of their Chances?

PROBLEM V.

*A* and *B* taking 12 Pieces of Money each, play with 3 Dice on this Condition, That if the Number 11 is thrown, *A* shall give *B* one Piece, but if 14 shall be thrown, then *B* shall give one to *A*; and he shall win the Game that first gets all the Pieces of Money. And the Proportion of *A*'s Chance to *B*'s is found to be, as 244,140,625 to 282,429,536,481.

*F I N I S.*