The Eighth Annual

EAST COAST OPERATOR ALGEBRAS SYMPOSIUM

Abstracts of Invited Talks

Dietmar Bisch

Title: Subfactors with composite Jones index

Abstract: The technique of composing two subfactors, pioneered by Haagerup and myself, has led to a rich collection of standard invariants (or planar algebras) of subfactors. I will report on some recent constructions and planar algebras obtained using this idea.

Jonathan Block

Title: Geometric structures and operator algebras

Abstract: We discuss a KK-like framework which we have developed to answer questions in geometry, topology and mathematical physics.

Richard Burstein

Title: Composition of Group-Subgroup Subfactors

Abstract: Following results of Jones (and Popa) establishing an equivalence between subfactor planar algebras and extremal subfactors, there has been considerable interest in constructing planar algebras abstractly. I will describe an abstract construction of a class of group-like subfactor planar algebras, obtained by taking fixed points of a graph planar algebra under a group of automorphisms. I will show that the subfactors corresponding to these planar algebras are obtained by composition of group-subgroup subfactors. This result generalizes the group-type subfactor construction of Bisch and Haagerup.

Caleb Eckhardt

Title: A noncommutative Perron-Frobenius operator

Abstract: Perron-Frobenius operators arise in the analysis of non-invertible measurable transforms. In certain situations, these operators can be studied from a topological point-of-view, and can realize nice extensions to noncommutative C*-algebras. In this talk we will focus on the Perron-Frobenius operator associated with Gauss's map on continued fractions expansions of numbers in [0,1], and it's extension to the so-called "noncommutative [0,1]."

Nigel Higson

Title: The Weyl Character Formula in KK-Theory

Abstract: Weyl's formula describes the characters of the irreducible representations of compact connected Lie groups. It is closely related to topological K-theory and index theory, as was pointed out by Atiyah and Bott a long time ago. I shall take another look at the topic, this time from the perspective of KK-theory and the Baum-Connes conjecture, in an effort to develop connections between the Baum-Connes conjecture and geometric representation theory. This is joint work with Jonathan Block and part of a larger project with Jonathan Block, David Ben-Zvi and David Nadler.

N. P. Landsman

Title: Functoriality of quantization: a KK-theoretic approach

Abstract: It is an old dream that quantization is functorial from some category of classical data to some category of quantum data. I proposed a specific operator-algebraic way of doing this in 2003, and showed that the "quantization commutes with reduction" conjecture of Guillemin and Sternberg (renowned in symplectic geometry but alas ignored in noncommutative geometry) then becomes a special case of the hypothetical functoriality of quantization (arXiv:math-ph/0307059). In fact, whilst the Guillemin-Sternberg conjecture as originally formulated only made sense for compact groups acting on compact spaces, the operator-algebraic setting seamlessly generalizes the conjecture to the noncompact case, where it essentially becomes a problem in equivariant index theory akin to the Baum-Connes conjecture. With my PhD student Peter Hochs, I proved a special case of the ensuing "noncompact" Guillemin-Sternberg conjecture (arXiv:math-ph/0512022), and more recently it was proved in great generality by Varghese Mathai and Weiping Zhang (arXiv:0806.3138). The aim of the talk is to give an overview of this entire development.

Emily Peters

TITLE: Classifying subfactors up to index five

ABSTRACT: In 1991 Haagerup began the project of studying subfactors with index "just a little higher" than four. The classification of subfactors with index up to $3+\sqrt{3}$ has recently been completed, and substantial progress has been made on the classification of subfactors with index less than 5. I'll talk about recent work with Scott Morrison, Noah Snyder and Dave Penneys in this direction.

Junhao Shen

TITLE: Amalgamated free products of C*-algebras with the MF property

ABSTRACT: Recall that a separable C*-algebra $\mathcal{A}$ is residually finite dimensional (RFD) if there is an embedding from $\mathcal{A}$ into $\prod_k \mathcal{M}_{n_k}(\mathbf{C})$ for a sequence of matrix algebras $\{\mathcal{M}_{n_k}(\mathbf{C})\}_{k=1}^{\infty}$. A separable C*-algebra $\mathcal{B}$ has the MF property (or $\mathcal{B}$ is an MF algebra) if there is an embedding from $\mathcal{B}$ into $\prod_k \mathcal{M}_{n_k}(\mathbf{C})/\sum_k \mathcal{M}_{n_k}(\mathbf{C})$ for a sequence of matrix algebras $\{\mathcal{M}_{n_k}(\mathbf{C})\}_{k=1}^{\infty}$.

Case of full amalgamated free products: We will show that full amalgamated free products of residually finite dimensional C*-algebras over finite dimensional C*-subalgebras, under a natural condition, are again residually finite dimensional. Similarly, full amalgamated free products of MF algebras over finite dimensional C*-subalgebras, under a natural condition, have the MF property.

Case of reduced amalgamated free products: We show that reduced amalgamated free products of UHF algebras over finite dimensional C*-subalgebras, under a natural condition, have the MF property.

In the end, we will talk about applications of our results.

It is joint work with Q. Li.

John Skukalek

TITLE: The Higson-Mackey Analogy for Complex Semisimple Groups and their Finite Extensions

ABSTRACT: In an article published in 1975, Mackey explored the relationship between the representation theories of semisimple Lie groups and semidirect products known as Cartan motion groups. Thirty years later, Higson showed that Mackey's "analogy" provides a representation-theoretic approach to the Baum-Connes conjecture in the case of complex semisimple groups. In this talk, I'll describe Higson's results and how they can be generalized to the almost connected scenario using Green's notion of twisted crossed product.

Andrew Toms

TITLE: Borel cardinality and classification of C*-algebras

ABSTRACT: Descriptive set theorists have over the past 25 years or so developed a theory of cardinality for Borel equivalence relations, allowing one to quantify how difficult it is to assign invariants to equivalence classes. We explore this theory in the context of separable C*-algebras, and show that isomorphism of simple nuclear algebras is at least as complex as Hjorth's notion of turbulence, and yet less complex than the orbit equivalence relation of a Polish group action. The usual classification theory of simple nuclear C*-algebras plays a critical role in both results. This is joint work with Ilijas Farah and Asger Tornquist.

David Vogan

TITLE: Calculating signatures and classifying unitary representations

ABSTRACT: In the early 1950s, Harish-Chandra showed that the C^* algebra of a simple Lie group G is *liminaire*: its image in any irreducible representation (precisely, a star-representation on a Hilbert space) consists exactly of the compact operators. Consequently unitary representations of G admit very well-behaved decompositions using the set $\widehat{G}_{\text{unit}}$ (the *unitary dual*) of irreducible unitary representations. Determining $\widehat{G}_{\text{unit}}$ is therefore a basic problem for harmonic analysis.

By the 1960s, Harish-Chandra had determined completely the irreducible representations $\widehat{G}_{\text{temp}}$ (the *tempered dual*) appearing in the decomposition of the regular representation on $L^2(G)$. Roughly speaking, he showed that

$\widehat{G}_{\mathrm{temp}}$ is a countable union of real vector spaces $V_i(\mathbf{R})$, each divided by the action of some finite group F_i.

Meanwhile Harish-Chandra had introduced a larger class $\widehat{G}_{\mathrm{adm}}$ (the *admissible dual*) of irreducible representations: no longer on Hilbert spaces, and without the fundamental requirement of preserving adjoints. Langlands proved that $\widehat{G}_{\mathrm{adm}}$ is the countable union of complex vector spaces $V_i(\mathbf{C})$ (the complexification of $V_i(\mathbf{R})$), each divided by the action of the same finite group F_i. This makes the admissible dual a beautiful "complexification" of the tempered dual. The unitary dual sits somewhere between...

In the 1970s, Knapp and his collaborators considered the set $\widehat{G}_{\mathrm{herm}}$ of irreducible Hermitian representations: star-representations on a possibly indefinite "Hilbert" space. They proved that

$$\widehat{G}_{\mathrm{herm}} = \text{union of real points of } V_i(\mathbf{C})/F_i.$$

(The point is that the finite group F_i makes this set of real points larger than $V_i(\mathbf{R})/F_i$.) We therefore have a picture

$$\widehat{G}_{\mathrm{temp}} \subset \widehat{G}_{\mathrm{unit}} \subset \widehat{G}_{\mathrm{herm}} \subset \widehat{G}_{\mathrm{adm}}$$

and each piece *except* the unitary dual has a simple description.

I will describe joint work with Jeff Adams, Marc van Leeuwen, Peter Trapa, and Wai Ling Yee that begins with any irreducible Hermitian representation and calculates the signature of the indefinite Hilbert space in which it acts. In particular, one can calculate whether the signature is definite, and therefore whether the representation is unitary.

The nature of the calculation is to begin in the tempered dual, corresponding to real vectors in $V_i(\mathbf{R})$; there the inner products are positive by Harish-Chandra's realization of them in $L^2(G)$. Then we deform into the imaginary directions in $V_i(\mathbf{C})$. Signature changes can happen only when the representations become reducible; and this reducibility is described in great detail by the Kazhdan-Lusztig theory.

Charlotte Wahl

Title: Invariants from noncommutative index theory for homotopy equivalences

Abstract: Using higher index theory for the signature operator we define for any homotopy equivalence between closed manifolds a noncommutative differential form which pairs with certain cyclic cocycles on the group algebra of the fundamental group. We show that after dividing out forms "localized at the identity" this invariant (called higher rho-invariant) is well-defined on the surgery structure set. We explain its relation to the classical rho-invariants of Atiyah-Patodi-Singer and of Cheeger-Gromov and give a formula describing its behavior under Cartesian products.

Robert Yuncken

Title: Pseudodifferential analysis on multiply-fibred manifolds

Abstract: Thanks to Connes, we know that the correct analytic structure for studying longitudinal operators on foliated manifolds — and in particular, on fibre bundles — is the C^*-algebra of the foliation groupoid. In representation theory, however, one encounters manifolds which admit numerous fibrations over different bases, and longitudinal pseudodifferential operators along each of them. In this talk I will discuss the compatibility between the groupoid C^*-algebras for such multiply-fibred manifolds, and some applications to index theory.
