Wild branching and tame expectations

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Abstract

Branched covers are open surjections of compact Hausdorff spaces with an upper bound on the cardinalities of the point preimages. The notion formalizes the intuition of maps misbehaving along small subsets of their domains, where derivatives are "more singular than generically" (the squaring self-map on the unit disk in the complex plane is a readily available example: branching occurs at the origin). Motivated by the classical concept, and making the non-commutative topologist's familiar move of rephrasing plain topology in C^*-algebraic terms, Pavlov and Troitskii define a non-commutative (or quantum) branched cover as an embedding of C^*-algebras admitting a finite-index expectation.

The two resulting classically competing definitions turn out to almost be equivalent, and the 'almost' is not without some interest of its own: classical branched covers $Y \to X$ produce C^* embeddings $C(X) \to C(Y)$ with finite-index expectations under appropriate good behavior conditions (e.g. X is metrizable), but there are examples where the sizes of the point preimages of the branched cover, roughly speaking, oscillate wildly enough to preclude such expectations. I will discuss such examples and analogous problems in the half-quantum case where X is still a plain compact Hausdorff space but one substitutes a possibly non-commutative C^*-algebra for $C(Y)$.