MATH 251: ABSTRACT ALGEBRA I
EXAM #1

Name ______________________________

Problem	Score
1	
2	
3	
4	
5	

Total ____________

Date: 8 October 2007.

Problem 1.

(a) Compute the inverse of 15 in the group $(\mathbb{Z}/127\mathbb{Z})^{\times}$.

(b) Determine if the group $(\mathbb{Z}/12\mathbb{Z})^{\times}$ is cyclic. Justify your answer.

Problem 2. Let

$$H = \left\{ \begin{pmatrix} a & -b \\ b & a \end{pmatrix} : a, b \in \mathbb{R} \text{ not both zero} \right\} \subset M_2(\mathbb{R}).$$

(a) Show that $H \subset GL_2(\mathbb{R})$.

(b) Show that H is a subgroup of $GL_2(\mathbb{R})$.

Problem 3.

(a) What is the largest order of an element $\sigma \in S_5$?

(b) Show that the groups D_{60} and S_5 are not isomorphic.

Problem 4. Let G be a group. Let $a, g \in G$, and suppose that a has order $n \in \mathbb{Z}_{>0}$. Prove that gag^{-1} has order n.

Problem 5. A group G is called *triplic* if for all $x \in G$, we have $x^3 = 1$.

Let $\phi : G \to H$ be an injective homomorphism of groups. Show that if H is triplic, then G is triplic.