MATH 251: ABSTRACT ALGEBRA I
EXAM #3

Name ____

Problem	Score
1	
2	
3	
4	
5	

Total ______________

Date: 14 December 2007.

Problem 1. Let F be a field. For a polynomial $f(x) = \sum_{i=0}^{n} a_i x^i \in F[x]$, we denote by

$$f'(x) = \sum_{i=1}^{n} i a_i x^{i-1} = a_1 + 2a_2 x + \cdots + n a_n x^{n-1}$$

the formal derivative of f.

(a) Let

$$I = \{f(x) \in F[x] : f(1) = f'(1) = 0\}.$$

Show that I is an ideal of $F[x]$.

(b) Let

$$J = \{f(x) \in F[x] : f(1) = f'(0) = 0\}.$$

Is J an ideal of $F[x]$? Prove or disprove.

Problem 2. Determine explicitly if the matrix

$$A = \begin{pmatrix} 1 & 5 \\ 3 & 2 \end{pmatrix} \in M_2(\mathbb{Z}/26\mathbb{Z})$$

is a zerodivisor.

Problem 3. Let R be a ring.

(a) Let $a \in R$ and suppose that $a^n \in R^\times$ for some $n \in \mathbb{Z}_{>0}$. Show that $a \in R^\times$.

(b) Suppose that $a^2 = 1$ for all $a \in R$ with $a \neq 0$. Show that R is a field.

Problem 4. Let R be an integral domain, and let $a, b \in R$. Prove that $(a) = (b)$ if and only if $a = ub$ for some $u \in R^{\times}$.

Problem 5. Show that the ideal of $\mathbb{Z}[i]$ generated by $2 + i$ is maximal.