MATH 251: ABSTRACT ALGEBRA I
REVIEW, EXAM #3

Problem 1. Prove that if R is an integral domain and $x^2 = 1$ for some $x \in R$ then $x = \pm 1$.

Problem 2. Let R be a ring. An element $a \in R$ is said to have a *left inverse* if $ba = 1$ for some $b \in R$. Let $a \in R$, and suppose that a has a unique left inverse b. Show that $ab = 1$, so that b is also a *right inverse*.

Problem 3. Let R be a commutative ring. An element $x \in R$ is said to be *nilpotent* if $x^n = 0$ for some $n \in \mathbb{Z}_{>0}$. Show that the set of all nilpotent elements in R forms an ideal.

Problem 4. Let R be the ring $R = \{a + bi + cj + dk : a, b, c, d \in \mathbb{Z}/3\mathbb{Z}\}$ where $i^2 = j^2 = (ij)^2 = -1$. Exhibit a zerodivisor in R.

Problem 5. Let F be a field, let $R = M_2(F)$, and suppose that I is an ideal of R. Show that $I = (0)$ or $I = R$. *[Note: $M_2(R)$ is not commutative.]*

Problem 6. Let $\phi : R \to S$ be a ring homomorphism. Let J be an ideal in S. Show that $\phi^{-1}(J)$ is an ideal of R containing $\ker \phi$.

Date: 5 December 2007; exam 14 December 2007.