MATH 251: ABSTRACT ALGEBRA I
WORKSHEET, DAY #11

Problem 1. Determine if each of the following is a homomorphism, an isomorphism, or neither. Justify your answer!

(a) The map

$$\begin{aligned} \operatorname{sgn} : \mathbb{R}^\times &\to \{1, -1\} \\ x \mapsto \operatorname{sgn}(x) &= \begin{cases} 1, & \text{if } x > 0; \\ -1, & \text{if } x < 0. \end{cases} \end{aligned}$$

(b) For F a field, the map $\det : GL_n(F) \to F^\times$.

(c) For $n \in \mathbb{Z}_{>0}$, the map $\phi : \mathbb{Z} \to \mathbb{Z}/n\mathbb{Z}$ where $\phi(a) = \overline{a}$ (the residue class of a modulo n).

(d) For G a group, the map $\phi : G \to G$ defined by $\phi(a) = a^{-1}$ for $a \in G$.

(e) For G an abelian group and $n \in \mathbb{Z}$, the map $\phi : G \to G$ defined by $\phi(a) = a^n$.

Problem 2. Let $\phi : G \to H$ be a homomorphism of groups. Prove that if ϕ is surjective and G is abelian, then H is abelian. Prove that if ϕ is injective and H is abelian, then G is abelian. (What happens if ϕ is just a homomorphism?)

Date: Friday, 21 September 2007.