Problem 1.

(a) Evaluate the integral
\[ \int (4x^3 + x^2) \, dx. \]

(b) Evaluate the integral
\[ \int x \left( x^{1.5} - x^{-1} + \frac{3}{x^2} \right) \, dx. \]

Solution. For (a), we have
\[ \int (4x^3 + x^2) \, dx = x^4 + \frac{x^3}{3} + C. \]

For (b), we multiply out to obtain
\[ \int x \left( x^{1.5} - x^{-1} + \frac{3}{x^2} \right) \, dx = \int \left( x^{2.5} + 3 - x + \frac{3}{x} \right) \, dx = \frac{x^{3.5}}{3.5} - x + 3 \ln |x| + C = \frac{2}{7} x^{3.5} - x + 3 \ln |x| + C. \]

Problem 2. The slope of the function \( f(x) \) at the point \((x, f(x))\) is equal to \( 9 - e^x \) and \( f(0) = 1 \). Find the function \( f(x) \).

Solution. We are given that \( f'(x) = 9 - e^x \), since the derivative is the slope, so \( f(x) = 9x - e^x + C \). Since \( f(0) = -1 + C = 1 \), we have \( C = 2 \), so \( f(x) = 9x - e^x + 2 \).

Problem 3. Evaluate the integral
\[ \int \frac{x^2}{(x^3 - 7)^{0.7}} \, dx. \]

Solution. We make the substitution \( u = x^3 - 7 \), so \( du = (3x^2) \, dx \) or \( x^2 \, dx = du/3 \). Then
\[ \int \frac{x^2}{(x^3 - 7)^{0.7}} \, dx = \int \frac{1}{u^{0.7}} \frac{du}{3} = \frac{1}{3} \int u^{-0.7} \, du = \frac{1}{3} \frac{u^{0.3}}{0.3} + C = \frac{10}{9} (x^3 - 7)^{0.3} + C. \]

Problem 4. Evaluate the integral
\[ \int (2x - 3)e^{2x^2-6x} \, dx. \]

Solution. We make the substitution \( u = 2x^2 - 6x \), so that \( du = (4x - 6) \, dx = 2(2x - 3) \, dx \), or \( (2x - 3) \, dx = du/2 \). Thus
\[ \int (2x - 3)e^{2x^2-6x} \, dx = \int e^u \frac{du}{2} = \frac{1}{2} e^u + C = \frac{1}{2} e^{2x^2-6x} + C. \]

Problem 5. Use the following graph of \( f(x) \) to compute \( \int_0^5 f(x) \, dx \).
Solution. The integral is just the area under the curve, which is $1/2 + 1 + 2 + 1 + 1 = 11/2$.

Problem 6(a). Calculate the left Riemann sum to approximate $\int_0^3 \frac{1}{1 + 2x} \, dx$ using $n = 3$ subdivisions.

Solution. Let $f(x) = 1/(1 + 2x)$. We have $a = 3$ and $b = 0$ so $\Delta x = (3 - 0)/3 = 1$. Thus the Riemann sum is simply

$$1 (f(0) + f(1) + f(2)) = 1 + 1/3 + 1/5 = 23/15.$$ 

Problem 6(b). Draw the rectangles representing the left Riemann sum for the following function $f(x)$ on the interval $[0, 3]$ using 6 subdivisions.

Problem 7. Compute the area under the graph of $f(x) = x(x^2 - 1)^4$ between $x = 0$ and $x = 1$.

Solution. We need to compute

$$\int_0^1 x(x^2 - 1)^4 \, dx.$$ 

We make the substitution $u = x^2 - 1$, so that $du = 2x \, dx$. If $x = 0$ then $u = -1$ and if $x = 1$ then $u = 0$. Thus

$$\int_0^1 x(x^2 - 1)^4 \, dx = \int_{-1}^0 u^4 \frac{du}{2} = \frac{u^5}{10}\bigg|_{-1}^0 = -(-1)^5/10 = 1/10.$$ 

Problem 8. Evaluate the definite integral

$$\int_1^e \left( 2x + \frac{2}{x} \right) \, dx.$$ 

Solution. We have

$$\int_1^e \left( 2x + \frac{2}{x} \right) \, dx = \left( x^2 + 2 \ln |x| \right) \bigg|_1^e = (e^2 + 2) - (1 + 0) = e^2 + 1.$$ 

Problem 9. A book publisher declares that the marginal cost to produce $x$ books is

$$C'(x) = 10 - \frac{500}{(x + 1)^3}$$
dollars, and that the fixed cost is 500 dollars. What is the cost function $C(x)$?

Solution. The marginal cost is the derivative of the total cost, so

$$C(x) = \int \left( 10 - \frac{500}{(x + 1)^3} \right) \, dx = 10x - 500 \int \frac{x}{(x + 1)^3} \, dx.$$  

We now make the substitution $u = x + 1$, so $du = dx$ and $x = u - 1$. Thus

$$\int \frac{x}{(x + 1)^3} \, dx = \int \frac{u - 1}{u^3} \, du = \int (u^{-2} - u^{-3}) \, du = -\frac{1}{u^2} + \frac{1}{u} + K = -\frac{1}{x + 1} + \frac{1}{2(x + 1)^2} + K.$$  

So

$$C(x) = 10x + 500 \left( -\frac{1}{x + 1} + \frac{1}{2(x + 1)^2} \right) + K.$$  

The fixed cost is $C(0) = 500(1/2) + K = 250 + K = 500$, so $K = 250$. Thus

$$C(x) = 10x + 500 \left( -\frac{1}{x + 1} + \frac{1}{2(x + 1)^2} \right) + 250.$$