MATH 20C: FUNDAMENTALS OF CALCULUS II
QUIZ #3

Problem 1. Evaluate the integral

$$\int 5x^2 e^{-x}\, dx.$$

Solution. We take $u = 5x^2$ and $v = e^{-x}$, since then u becomes simpler upon differentiation. So:

$$\begin{array}{c|cc} & D & I \\ \hline + & 5x^2 & e^{-x} \\ - & 10x & -e^{-x} \\ +\int & 10 & e^{-x} \end{array}.$$

Recall $\int e^{-x} = -e^{-x} + C$, by substituting $u = -x$ or by the rule we have learned. Thus

$$\begin{aligned} \int 5x^2 e^{-x}\, dx &= 5x^2(-e^{-x}) - 10x(e^{-x}) + \int 10e^{-x}\, dx \\ &= -5x^2 e^{-x} - 10xe^{-x} - 10e^{-x} + C \\ &= -(5x^2 + 10x + 10)e^{-x} + C. \end{aligned}$$

Problem 2. Evaluate the integral

$$\int_1^4 x^{1/2} \ln x\, dx.$$

Solution. We first compute the antiderivative, then apply the Fundamental Theorem of Calculus:

$$\begin{array}{c|lc} & D & I \\ \hline + & \ln x & x^{1/2} \\ -\int & 1/x & x^{3/2}/(3/2) = 2/3x^{3/2} \end{array}.$$

So

$$\begin{aligned} \int x^{1/2} \ln x\, dx &= \ln x \cdot \frac{2}{3}x^{3/2} - \int \frac{1}{x} \cdot \frac{2}{3}x^{3/2}\, dx \\ &= \frac{2}{3}x^{3/2} \ln x - \frac{2}{3} \int x^{1/2}\, dx \\ &= \frac{2}{3}x^{3/2} \ln x - \frac{2}{3} \frac{x^{3/2}}{3/2} + C \\ &= \frac{2}{3}x^{3/2} \ln x - \frac{4}{9}x^{3/2} + C \\ &= x^{3/2} \left(\frac{2}{3} \ln x - \frac{4}{9} \right) + C. \end{aligned}$$

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Therefore we can now evaluate the definite integral:

$$\begin{aligned}\int_1^4 x^{1/2} \ln x \, dx &= \left(\frac{2}{3} x^{3/2} \ln x - \frac{4}{9} x^{3/2} \right) \Big|_1^4 \\ &= \frac{2}{3} 4^{3/2} \ln 4 - \frac{4}{9} 4^{3/2} - \left(\frac{2}{3} 1^{3/2} \ln 1 - \frac{4}{9} 1^{3/2} \right) \\ &= \frac{16}{3} \ln 4 - \frac{32}{9} + \frac{4}{9} = \frac{16}{3} \ln 4 - \frac{28}{9}.\end{aligned}$$

Note $4^{3/2} = 8$. Fun huh?