Problem 1. Find the total value of the income stream \( R(t) = 40000 \) on the interval \( 0 \leq t \leq 5 \) and find its future value at the end of the interval using the interest rate 10%.

Solution. We have the total value is \((a = 0, b = 5)\)

\[
TV = \int_{a}^{b} R(t) \, dt = \int_{0}^{5} 40000 \, dt = 40000t \bigg|_{0}^{5} = 200000.
\]

To compute the future value, we have \( r = 0.1, \) so

\[
FV = \int_{a}^{b} R(t)e^{r(b-t)} \, dt = \int_{0}^{5} 40000e^{0.1(5-t)} \, dt = -\frac{40000}{0.1}e^{0.1(5-t)} \bigg|_{0}^{5} = 259488.51.
\]

Problem 2. Find the total value of the income stream \( R(t) = 50000 + 2000t \) on the interval \( 0 \leq t \leq 10 \) and find its present value at the beginning of the interval using the interest rate 5%.

Solution. The total value is

\[
TV = \int_{0}^{10} (50000 + 2000t) \, dt = (50000t + 1000t^2) \bigg|_{0}^{10} = 600000.
\]

The present value is \((r = 0.05)\)

\[
PV = \int_{a}^{b} R(t)e^{r(a-t)} \, dt = \int_{0}^{10} (50000 + 2000t)e^{-0.05t} \, dt.
\]

Use integration by parts with \( u = 50000 + 2000t \) and \( v = e^{-0.05t} \) to obtain

\[
\int_{0}^{10} (50000 + 2000t)e^{-0.05t} \, dt = (\frac{-1000000 + 40000t}{0.05}e^{-0.05t} - 800000e^{-0.05t}) \bigg|_{0}^{10} = 465632.55.
\]

Problem 3. You begin saving for your retirement by investing $700 per month in an annuity with a guaranteed interest rate of 6% per year. You increase the amount you invest at the rate of 3% per year. With continuous investment and compounding, how much will you have accumulated in the annuity by the time you retire in 45 years?

Solution. The revenue stream is \( R(t) = 12 \times 700e^{0.03t} = 8400e^{0.03t} \) since you do it for each month. So the future value is

\[
\int_{0}^{45} (8400e^{0.03t})e^{0.06(45-t)} \, dt = 8400e^{2.7} \int_{0}^{45} e^{-0.03t} \, dt = -280000e^{2.7}e^{-0.03t} \bigg|_{0}^{45} = 3086245.73.
\]

See why you should start saving now?

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