MATH 241: ANALYSIS IN SEVERAL REAL VARIABLES I
REVIEW SESSION, EXAM #2

(1) Show that $f(x) = x/(x^2+1)$ is uniformly continuous on $\mathbb{R}$.

(2) Let

$$f : \mathbb{R} \to \mathbb{R}$$
$$f(x) = \begin{cases} e^{-1/x}, & \text{if } x \neq 0; \\ 1, & \text{if } x = 0. \end{cases}$$

Is f differentiable at $x = 0$?

(3) Is there a continuous function $f : \mathbb{R} \to \mathbb{R}$ such that $f(\mathbb{R})$ consists of two points?

(4) Let $f : A \to \mathbb{R}$ be continuous and suppose that $f(x) \geq 0$ for all $x \in A$. Let $g : A \to \mathbb{R}$ be defined by $g(x) = \sqrt{f(x)}$ for $x \in A$. If f is continuous at $c \in A$, show that g is continuous at c.

(5) Let A be a bounded set. Show that $\overline{A}$ is bounded.

(6) Let A be an open set and let $c \in A$. Suppose that $f : A \to \mathbb{R}$ is differentiable. Suppose further that $\lim_{x\to c} f'(x)$ exists. Show that $f'(c) = \lim_{x\to c} f'(x)$.

(7) Let A be a bounded set and let $f : A \to \mathbb{R}$ be uniformly continuous. Show that f is bounded.

(8) Let $f, g : A \to \mathbb{R}$ be continuous functions. Define the function $h : A \to \mathbb{R}$ by $h(x) = \max(f(x), g(x))$. Show that h is continuous.

(9) Let $A, B \subset \mathbb{R}$. Show that $\overline{A \cup B} = \overline{A} \cup \overline{B}$.

(10) Let f be differentiable on $\mathbb{R}$. Suppose that $f(0) = 0$ and that $1 \leq f'(x) \leq 2$ for all $x \geq 0$. Prove that $x \leq f(x) \leq 2x$ for all $x \geq 0$.

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