

**MATH 241: ANALYSIS IN SEVERAL REAL VARIABLES I  
PROOF PARTY**

**Problem 1.** Let  $x \in \mathbb{Q}$  and let  $y \in \mathbb{R}$  with  $y \notin \mathbb{Q}$ . Prove that if  $x \neq 0$  then  $xy \notin \mathbb{Q}$ .

**Problem 2.** Let  $A, B$  be nonempty subsets of  $\mathbb{R}$  with  $A \subseteq B$ . Suppose that  $B$  is bounded above. Prove that  $\sup A \leq \sup B$ .

**Problem 3.** Prove that  $n^2 \leq n!$  for all  $n \geq 4$ .

**Problem 4.** Let  $A, B$  be sets. Suppose that  $B$  is uncountable and that there exists a surjective map  $f : A \rightarrow B$ . Prove that  $A$  is uncountable.