MATH 251: ABSTRACT ALGEBRA I
EXAM #2

Name _____

Problem	Score
1	
2	
3	
4	

Total _____

Date: 2 November 2011.

Problem 1.

(a) Let $G = Z_{132} = \langle x \rangle$ be the cyclic group of order 132 generated by x. For an integer n, let $H_n = \langle x^n \rangle \leq G$. Compute the order of H_n. Is H_{38} contained in H_{110}, or vice versa?

(b) Let G be a group with $\#G = 117$ and $H \trianglelefteq G$ a normal subgroup with $\#H = 9$. Show that G/H is abelian.

Problem 2. Let G be a group, let $N \leq G$, and let $\phi : G \to H$ be a homomorphism.

(a) Prove that the image $\phi(N)$ is a subgroup of H.

(b) Suppose that $N \trianglelefteq G$ and that ϕ is surjective. Show that $\phi(N) \trianglelefteq H$.

Problem 3. Let G be a group and let $a \in G$ satisfy $a^2 = 1$. Suppose that $\langle a \rangle \trianglelefteq G$. Prove that $a \in Z(G)$.

Problem 4.

(a) What is the sign of the permutation $\sigma = (1\ 3\ 5\ 7\ 9)(2\ 4\ 6)(11\ 15)$?

(b) Draw the lattice of subgroups of S_3.

(c) Show that if $n \geq 3$ then every element in A_n is a product of 3-cycles.