MATH 052: INTRODUCTION TO PROOFS
EXAM #2

Name ____

Problem	Score
1	
2	
3	
4	
5	

Total ____________

Date: 2 November 2011.

Problem 1.

(a) Determine the power set $\mathcal{P}(A)$ of the set $A = \{0, 2, -6\}$.

(b) Consider the following subsets of $A = \{1, 2, 3, 4, 5, 6\}$:

$S_1 = \{\{1, 3, 6\}, \{2, 4\}, \{5\}\}$ $\quad$ $S_2 = \{\{1, 2, 3\}, \{4\}, \emptyset, \{5, 6\}\}$

$S_3 = \{\{1, 2\}, \{3, 4, 5\}, \{5, 6\}\}$ $\quad$ $S_4 = \{\{1, 4\}, \{3, 5\}, \{2\}\}$

Determine which of these sets are partitions of A.

(c) True or false: if $f : A \to B$ is injective and $g : B \to C$ is injective, then $g \circ f : A \to C$ is injective.

(d) Give an example of a function $f : A \to B$ which is surjective but not injective.

(e) Evaluate the sum $\sum_{k=1}^{4} (k^2 - 2)$.

Problem 2. Prove by induction that

$$1 + 2 + 2^2 + \cdots + 2^{n-1} = 2^n - 1$$

for all integers $n \geq 1$.

Problem 3.

(a) Recall that $n \in \mathbb{Z}$ is *even* if $n = 2k$ for some $k \in \mathbb{Z}$. Consider the relation R defined on $\mathbb{Z}$ by aRb if and only if $a+b$ is even. Which of the properties reflexive, symmetric, and transitive does the relation R possess? Justify your answers.

(b) Let $A = \{1, 2, 3, 4, 5\}$. The relation

$$R = \{(1,1), (1,5), (2,2), (2,3), (3,2), (3,3), (4,4), (5,1), (5,5)\}$$

is an equivalence relation on A. Draw the graph associated to R and determine its equivalence classes.

Problem 4. Let A, B, C be sets. Prove that

$$A \times (B \cap C) = (A \times B) \cap (A \times C).$$

Problem 5. Show that the map

$$\begin{aligned} f : \mathbb{R} \setminus \{2\} &\to \mathbb{R} \setminus \{3\} \\ x &\mapsto f(x) = \frac{3x}{x-2} \end{aligned}$$

is a bijection.