MATH 052: INTRODUCTION TO PROOFS
HOMEWORK #18

Problem 2.9.1. For (a), we have the partitions of $\mathbb{R}$ given by

$$\{\mathbb{R}_{<0}, \{0\}, \mathbb{R}_{>0}\}, \quad \{\mathbb{Q}, \mathbb{R} \setminus \mathbb{Q}\}, \quad \{[n, n+1) : n \in \mathbb{Z}\}$$

where recall $[a, b) = \{x \in \mathbb{R} : a \leq x < b\}$ is the half-open interval. But answers will vary. For (b) we can partition people into

The set of people born in January, the set of people born in February, ..., the set of people born in December

or

The set of people with 10 fingers, the set of people with less than 10 fingers, the set of people with more than 10 fingers

and so on. For (c), we partition $\mathbb{Z}$ with the sets

$$\{\{n \in \mathbb{Z} : n \text{ is even}\}, \{n \in \mathbb{Z} : n \text{ is odd}\}\}, \quad \{\mathbb{Z}_{<0}, \{0\}, \mathbb{Z}_{>0}\}, \quad \{\{n\} : n \in \mathbb{Z}\},$$

for example.

Problem 2.9.2. A partition of this set of 5 elements into blocks that have at least two elements can only happen by taking the trivial partition $\{\{1, 2, 3, 4, 5\}\}$ or by taking a partition into blocks of size two and three:

$$\{\{1, 2\}, \{3, 4, 5\}\}, \quad \{\{1, 3\}, \{2, 4, 5\}\}, \quad \ldots, \quad \{\{4, 5\}, \{1, 2, 3\}\}$$

of which there are $5 \cdot 4/2 = 10$ possibilities.

Problem 2.9.3. Part (a) is not a partition because 4 is not in one of the subsets. Part (b) is not a partition because the emptyset $\emptyset$ is not allowed as a block. Part (c) is a partition: every element in $\{-1, 1, 2, 3, 4\}$ appears in exactly one subset. Finally, part (d) is not a partition because the sets are not disjoint: 1 appears in more than one block.

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