MATH/CS 295: CRYPTOGRAPHY
HOMEWORK #10 ADDITIONAL PROBLEMS

Problem 3.C. For $s \in [0,1]$ define $L_s : \mathbb{R}_{>e} \to \mathbb{R}$ (where $e = \exp(1)$) by

$$L_s(x) = \exp\left((\log x)^s (\log\log x)^{1-s}\right).$$

(a) Show that $L_0(x) = \log x$ and $L_1(x) = x$.

(b) Show that

$$L_s(x) \le L_t(x)$$

for all $x \in \mathbb{R}_{>e^e}$ whenever $s \le t$.

(c) Show that the function

$$L_{1/2}(x) = \exp(\sqrt{\log x \log\log x})$$

is *subexponential*: i.e., show that for every $\epsilon > 0$, we have

$$L_{1/2}(x) = O(x^\epsilon).$$

[Hint: Take the logarithm of both sides of the inequality $L_{1/2}(x) \le x^\epsilon$ and use l'Hôpital's rule.]