QUIZ #1: CALCULUS 1A (Stankova)
Wednesday, January 28, 2004
Section 10:00–11:00 (Voight)

Problem 1. *The position of a hydrogen fuel-cell vehicle is given by the values in the following table:*

t (seconds)	0	5	10	15	20	25
s (feet)	30	150	450	950	1600	2575

(a) *Find the average velocity for the time period beginning when* $t = 5$ *and lasting:*
  (i) 15 *seconds*
  (ii) 10 *seconds*
  (iii) 5 *seconds*
(b) *Use the graph of* s *as a function of* t *to estimate the instantaneous velocity when* $t = 5$.

Solution. The average velocity is given by the distance traveled divided by the amount of time. A time period beginning when $t = 5$ and *lasting* 15 seconds ends at $t = 20$ seconds. Therefore for (i), the average velocity over $[5, 20]$ is

$$v = \frac{s(20) - s(5)}{20 - 5} = \frac{1600 - 150}{15} = \frac{1450}{15} = \frac{290}{3} \text{ ft/s.}$$

[You could have left the fraction unsimplified!]

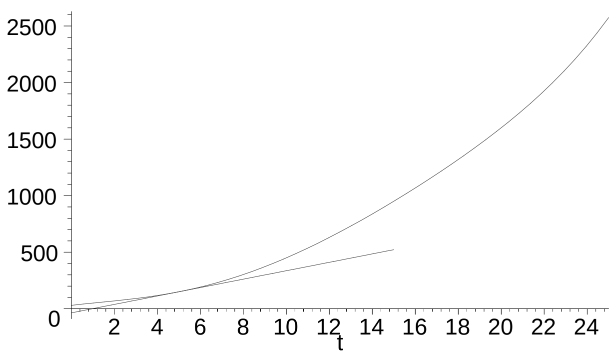
Similarly, for (ii) we have

$$\frac{950 - 150}{15 - 5} = \frac{800}{10} = 80 \text{ ft/s}$$

and for (iii) we have

$$\frac{450 - 150}{10 - 5} = \frac{300}{5} = 60 \text{ ft/s.}$$

For (b), we graph *with the proper scale* the points given in the graph as follows:



We fit a curve to the points $(0, 30), (5, 150), \ldots, (25, 2575)$.

The tangent line appears to go through the point $(15, 500)$, so we estimate its slope to be

$$\frac{500 - 150}{15 - 5} = 35 \text{ ft/s.}$$

The instantaneous velocity is exactly this slope. [It is acceptable to estimate the slope in other ways using the graph, as long as you justify your answer.]

QUIZ #1: CALCULUS 1A (Stankova)
Wednesday, January 28, 2004
Section 11:00–12:00 (Voight)

Problem 1. *If a pellet is shot upward by the Mars rover, its height in kilometers after t seconds is given by $h = 3t - t^2$.*

(a) *Find the average velocity over the given time intervals:*
  (i) $[0, 1]$
  (ii) $[0, 0.5]$
  (iii) $[0, 0.1]$

(b) *Graph the height of the pellet as a function of time; draw the tangent line to the graph at time $t = 0$.*

(c) *Find the initial velocity at which the pellet was shot.*

SOLUTION. The average velocity is given by the distance traveled divided by the amount of time. Therefore the average velocity for (i) is

$$v = \frac{h(1) - h(0)}{1 - 0} = (3(1) - 1^2) - 0 = 2 \text{ m/s}.$$

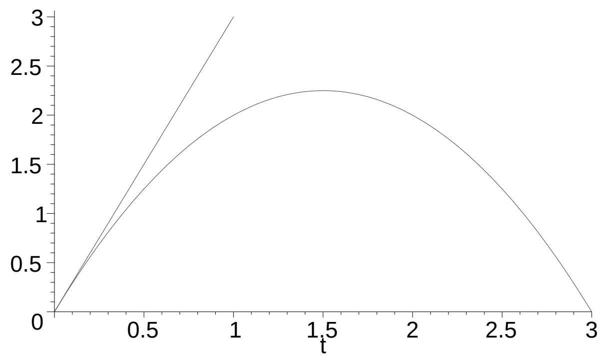
Similarly, for (ii) we have

$$v = \frac{3(0.5) - (0.5)^2 - 0}{0.5 - 0} = \frac{1.5 - 0.25}{.5} = 2(1.25) = 2.5 \text{ m/s}$$

and for (iii) we have

$$v = \frac{3(0.1) - (0.1)^2 - 0}{0.1 - 0} = \frac{0.3 - 0.01}{.1} = \frac{0.29}{.1} = 2.9 \text{ m/s}.$$

For (b), we plot points: we find that at $t = 0$ the height is $h = 0$, at $t = 1$ the height is $h = 2$, and so on:



For (c), notice that the slope of the tangent line, which is the same as the initial velocity or the instantaneous velocity at $t = 0$, is approximately 3 m/s. Also, the average velocities computed in (a) (none other than the slopes of the secant lines) approach this same value, 3 m/s.