MATH 255: ELEMENTARY NUMBER THEORY
EXAM #2

Name ____

Please complete the following problems in the space provided. Please include all relevant intermediate calculations and explain your work.

Problem 1.

(a) Find a root of the polynomial $x^5 + 10$ modulo 121.

(b)* How many roots does $x^5 + 10$ have modulo $11^4 = 14641$?

Problem 2. (Short answer.)

(a) How many primitive roots are there modulo the prime 257?

(b) Compute the Legendre symbol $\left(\frac{17}{47}\right)$.

(c) What are the last two decimal digits of 7^{642}?

(d) Let f be a multiplicative function with $f(1) = 0$. Show that $f(n) = 0$ for all n.

(e) If a is a quadratic residue modulo p, show that a is not a primitive root modulo p.

Problem 3. Show that $a^6 - 1$ is divisible by 168 whenever $\gcd(a, 42) = 1$.

Problem 4. Let n be a perfect number. Show that for all $k \in \mathbb{Z}_{\geq 2}$ that kn is abundant.

Problem 5. The integer $n = pq = 280171$ is used in an RSA cryptosystem. Through espionage, you determine that

$$\sigma(n) = 281232.$$

Find p and q.

Problem 6*. Let p be an odd prime and let r be a primitive root modulo p. Show that the order of $r + p$ modulo p^2 is either $p - 1$ or $p(p - 1)$.