

MATH 252: ABSTRACT ALGEBRA II
HOMEWORK #12

Problem 1. Let p be prime and define

$$a_n(p) = \#\{f \in \mathbb{F}_p[X] : \deg f = n, f \text{ monic irreducible}\}.$$

- (a) Show that $a_2(p) = (p^2 - p)/2$ and $a_3(p) = (p^3 - p)/3$.
(b) Use the equality

$$(*) \quad \sum_{d|n} da_d(p) = p^n$$

(which you may assume) to compute $a_n(2)$ for $n = 1, \dots, 5$.

- (c) Use (*) to prove that

$$\frac{p^n - 2p^{n/2}}{n} < a_n(p) \leq \frac{p^n}{n}.$$

Conclude that the probability that a random monic polynomial of degree n over \mathbb{F}_p is irreducible is roughly $1/n$.