

**MATH 351: RIEMANN SURFACES AND DESSINS D'ENFANTS
HOMEWORK #26**

Problem 26.1. Check by direct calculation that if $f(z) \in \mathbb{C}(z)$ is a rational function, then the meromorphic differential $f(z) dz$ on \mathbb{P}^1 satisfies the residue theorem. [*Hint: Use partial fractions!*]

Problem 26.2. Let $\Lambda \subseteq \mathbb{C}$ be a lattice and let $\pi : \mathbb{C} \rightarrow X = \mathbb{C}/\Lambda$ be the natural quotient map.

(a) Let $\lambda \in \Lambda$. Define the curve $r_\lambda : [0, 1] \rightarrow \mathbb{C}$ by $r_\lambda(t) = t\lambda$. Show that $\pi \circ r_\lambda$ is a closed path on X , and compute the integral $\int_{\pi \circ r_\lambda} dz$ on X .

(b) Show that

$$\left\{ \int_r dz : r \text{ a closed path on } X \right\} = \Lambda.$$