ERRATA:
A CANONICAL FORM FOR POSITIVE DEFINITE MATRICES

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This note gives errata for the article A canonical form for positive definite matrices [1].

1. Errata

(1) In (2.2.6), the characteristic vector set of $L_2$ (having Gram matrix $A_2$) should be lifted to $L$ using closest vectors with respect to $L_1$ instead. This is what was implemented in the code.

More precisely, Then (2.2.6) should read

$$V_{cv}(A) := B_1 V_{wr-cv}(A_1) \cup \bigcup_{v \in P_2 V_{cv}(A_2)} (v - B_1 CV(A_1, B_1^{-1}(v - \proj(v))))$$

where $B_1^{-1} = (B_1^T B_1)^{-1} B_1^T$ is the pseudo-inverse, a right inverse to $B_1$. This is the union of the well-rounded characteristic vector set for $L_1$ together with all vectors in the cosets $u_i + L_1$ with minimal distance to $L_1$, so is well-defined independent of the choice of lifts $u_i$.

To prove that this is a characteristic vector set, the argument in Theorem 2.2.7(b) should be replaced with the following.

We now prove (b), checking the conditions (i) and (ii). For part (i), by construction $B_1 V_{wr-cv}(A_1)$ spans $L_1 = \ker \proj |_L$ and by induction we have that $V_{cv}(A_2)$ spans $L_2$ so that $\text{span}(V_{cv}(A_2)) \subseteq L$ projects onto $L_2$ via $\proj$, so together they span $L$. Next we show that (ii) holds. First, the lattice $L_1$ spanned by minimal vectors is well-defined, independent of $U \in \text{GL}_n(\mathbb{Z})$, and $V_{wr-cv}(A_1)$ is a characteristic vector set. Hence too the projection $L_2$ is independent of $U$: by induction on the dimension, we know that $V_{cv}(A_2)$ is a characteristic vector set, and for each vector, the set of minimal vectors in each coset satisfies the necessary transformation property as in the case of $V_{wr-cv}$. So altogether, these form a characteristic vector set.

We also write this out in terms of (convenient) bases. Running the algorithm for $A$ and $A' = U^T A U$ with $U \in \text{GL}_n(\mathbb{Z})$, we may suppose that $v_i' = U^{-1} v_i$ and $w_i' = U^{-1} w_i$ by using the transformation property of $\text{Min}(A)$. Then $A_i' = A_i$ and $B_i' = U^{-1} B_i$ for $i = 1, 2$, and so we may further suppose that $w_i' = U^{-1} u_i$ so $P_2' = U^{-1} P_2$. We conclude by noting that CV also has the compatible transformation property: for all $v' \in P_2' V_{cv}(A_2')$, we
have
\[
v' - B_1' CV(A_1', (B_1')^{-1}(v' - \text{proj}'(v'))
= U^{-1}v - U^{-1}B_1 CV(A_1, B_1^{-1}U(U^{-1}v - U^{-1}\text{proj}(v)))
= U^{-1}v - U^{-1}B_1 CV(A_1, B_1^{-1}(v - \text{proj}(v))).
\]

References

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