ERRATA: *COMPUTING CLASSICAL MODULAR FORMS*

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This note gives errata and addenda for the article *Computing classical modular forms* [1].

1. Errata

(1) Theorem 5.1.2: Replace $S_k(\Gamma_1(n);\mathbb{Q})$ with $S_k(\Gamma_1(N);\mathbb{Q})$.

(2) Section 8.3: Replace

> For a newform f with trivial character, its Fricke eigenvalue is *minus* the sign of the functional equation of its L-function, and each W_q-eigenvalue is the sign of a certain local functional equation.

with

> For a newform of weight k and trivial character, the Fricke eigenvalue ϵ is related to the sign ε that appears in the functional equation (9.1.3) via $\epsilon = (-1)^{k/2}\varepsilon$, see Miyake [70, Cor. 4.3.7]. Each W_q-eigenvalue is similarly related to the sign of a certain local functional equation.

(3) Conjecture 8.5.1: Missing new subspace. Replace statement with

> For all $k \geq 2$, the space $S_k^{\mathrm{new}}(\Gamma_0(2))$ decomposes under the Atkin–Lehner operator W_2 into Hecke irreducible subspaces of dimensions $\lfloor d/2 \rfloor$ and $\lceil d/2 \rceil$ where $d := \dim_{\mathbb{C}} S_k^{\mathrm{new}}(\Gamma_0(2))$.

2. Addenda

(1) Section 8.5: Strike "However, we observed behavior analogous to the Maeda conjecture in weight 1 up to weight $k \leq 400$, with the additional prediction that the Atkin–Lehner operator splits the space as evenly as possible." Replace with "However, we observed behavior analogous to the Maeda conjecture in weight 1 up to weight $k \leq 400$. The Atkin–Lehner operator W_2 splits the space as evenly as possible, and the W_2-eigenspaces appear to always be irreducible."

(2) We can prove the observation following Conjecture 8.5.1. Replace the text up to Question 8.5.3 with the following:

> The dimensions in the corollary follow from Martin [Thm. 2.2] (Kimball Martin, *The basis problem revisited*, arXiv:1804.04234v2, 2019), which implies that for even weights $k > 2$ we have
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> $$\dim S_k^{\mathrm{new}}(\Gamma_0(2))^+ - \dim S_k^{\mathrm{new}}(\Gamma_0(2))^- = \begin{cases} 0 & k = 4, 6 \bmod 8, \\ (-1)^{k/2} & k \equiv 0, 2 \bmod 8, \end{cases}$$
>
> is is only the irreducibility of the eigenspaces that is conjectural. The factor $(-1)^{k/2}$ in (8.5.2) appears as 1 in [64, Thm. 2.2] because there the Atkin-Lehner operator follows the convention of Diamond–Shurman [39, p. 209], which includes a factor of $(-1)^{k/2}$, while we are following the convention of Miyake [70], which does not include this factor.
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> One can find similar formulas for $\dim S_k^{\mathrm{new}}(\Gamma_0(N))^+ - \dim S_k^{\mathrm{new}}(\Gamma_0(N))^-$ for any squarefree N in Martin, in which case they are a linear function of the class number $h(-4N)$. For general $N > 4$ not of the form $M^2, 2M^2, 3M^2, 4M^2$ with M squarefree, we refer the reader to Helfgott [pp. 266–267] (Harald A. Helfgott, *Root numbers and the parity problem*, Ph.D. Thesis, Princeton University, 2003).

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References

[1] Alex J. Best, Jonathan Bober, Andrew R. Booker, Edgar Costa, John Cremona, Maarten Derickx, Min Lee, David Lowry-Duda, David Roe, Andrew V. Sutherland, and John Voight, *Computing classical modular forms*, Arithmetic Geometry, Number Theory, and Computation, eds. Jennifer S. Balakrishnan, Noam Elkies, Brendan Hassett, Bjorn Poonen, Andrew V. Sutherland, and John Voight, Simons Symp., Springer, Cham, 2021, 131–213.