This note gives errata and addenda for the article Computing classical modular forms [1].

1. Errata

(1) Theorem 5.1.2: Replace $S_k(\Gamma_1(n); \mathbb{Q})$ with $S_k(\Gamma_1(N); \mathbb{Q})$.

(2) Section 8.3: Replace

For a newform $f$ with trivial character, its Fricke eigenvalue is \textit{minus} the sign of the functional equation of its $L$-function, and each $W_q$-eigenvalue is the sign of a certain local functional equation.

with

For a newform of weight $k$ and trivial character, the Fricke eigenvalue $\epsilon$ is related to the sign $\varepsilon$ that appears in the functional equation (9.1.3) via $\epsilon = (-1)^{k/2} \varepsilon$, see Miyake [70, Cor. 4.3.7]. Each $W_q$-eigenvalue is similarly related to the sign of a certain local functional equation.

(3) Conjecture 8.5.1: Missing new subspace. Replace statement with

For all $k \geq 2$, the space $S_{\text{new}}^k(\Gamma_0(2))$ decomposes under the Atkin–Lehner operator $W_2$ into Hecke irreducible subspaces of dimensions $\lfloor d/2 \rfloor$ and $\lceil d/2 \rceil$ where $d := \dim_{\mathbb{C}} S_{\text{new}}^k(\Gamma_0(2))$. In the discussion that follows, replace the reference Kimball [64] with the reference to [Kimball Martin, Refined dimensions of cusp forms, and equidistribution and bias of signs, J. Number Theory 188 (2018), 1–17].

2. Addenda

(1) Section 8.5: Strike “However, we observed behavior analogous to the Maeda conjecture in weight 1 up to weight $k \leq 400$, with the additional prediction that the Atkin–Lehner operator splits the space as evenly as possible.” Replace with “However, we observed behavior analogous to the Maeda conjecture in weight 1 up to weight $k \leq 400$. The Atkin–Lehner operator $W_2$ splits the space as evenly as possible, and the $W_2$-eigenspaces appear to always be irreducible.”

(2) We can prove the observation following Conjecture 8.5.1. Replace the text up to Question 8.5.3 with the following:

The dimensions in the corollary follow from Martin [Thm. 2.2] (Kimball Martin, The basis problem revisited, arXiv:1804.04234v2, 2019), which implies that for even weights $k > 2$ we have

$$\dim S_{\text{new}}^k(\Gamma_0(2))^+ - \dim S_{\text{new}}^k(\Gamma_0(2))^- = \begin{cases} 0 & k = 4, 6 \text{ mod } 8, \\ (-1)^{k/2} & k \equiv 0, 2 \text{ mod } 8, \end{cases}$$

is only the irreducibility of the eigenspaces that is conjectural. The factor $(-1)^{k/2}$ in (8.5.2) appears as 1 in [64, Thm. 2.2] because there the Atkin-Lehner operator follows the convention of Diamond–Shurman [39, p. 209], which includes a factor of $(-1)^{k/2}$, while we are following the convention of Miyake [70], which does not include this factor.

One can find similar formulas for $\dim S_{\text{new}}^k(\Gamma_0(N))^+ - \dim S_{\text{new}}^k(\Gamma_0(N))^-$ for any squarefree $N$ in Martin, in which case they are a linear function of the class number $h(-4N)$. For
general $N > 4$ not of the form $M^2, 2M^2, 3M^2, 4M^2$ with $M$ squarefree, we refer the reader to Helfgott [Harald A. Helfgott, *Root numbers and the parity problem*, Ph.D. Thesis, Princeton University, 2003, pp. 266-267].

References