ERRATA AND ADDENDA:
ON COMPUTING BELYI MAPS

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This note gives some errata and addenda for the article On computing Belyi maps [3]. The authors thank Sam Schiavone.

ERRATA

(1) Section 1, (D3): To be more precise, replace with

$O$ is a cyclic orientation of the edges around every vertex. That is, $O = (O_0, O_1)$ is a pair of permutations in Sym($E$) with the property that two edges $e, e'$ are in the same orbit under $O_0$ if and only if the vertices $v_0(e), v_0(e')$ colored 0 are equal, and the same with the orbit under $O_1$ for the vertices colored 1.

Then replace the first sentence with “Concretely, a cycle of $O_0$ with common vertex $v$ colored 0 describes the result of rotating the edges around $v$ counterclockwise, and the same with $O_1$. For some examples, see e.g. Couveignes–Granboulan [1, p. 15–16].”

(2) Section 2, “The ASD differentiation trick”: Elkies [2, Footnote 4] notes that the use of differentiation in this context goes back at least as far as 1956, appearing on the Putnam exam (and thus predating Atkin and Swinnerton-Dyer).

(3) Section 6, paragraph 3, “In particular, the CM factors of the Jacobian factors of the Galois Belyi curves are essentially known; they come from Fermat curves”: This is not correct; this only is only true for the factors of the Jacobian that are one-dimensional subrepresentations of the automorphism group, as happens in the case of abelian automorphism groups but not in general. In Section 6.5 of the article of Wolfart that is referred to, Hurwitz curves are constructed that are not of CM type and that therefore cannot come from Fermat curves.

(4) Section 8, Simplification: When considering denominator and numerator, make it clear that it is Belyi maps on hyperelliptic curves that are considered throughout.

ADDENDA

(1) Section 4, after (4.5): Elkies notes that one can often generate spaces of modular forms or more simply compute a Hauptmodul for congruence subgroups of SL$_2$(Z) (and the corresponding Belyi maps) by using other techniques, such as eta quotients or theta functions associated to integral quadratic forms, as well as their images under differentiation and Hecke operators.

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References

