ERRATA AND ADDENDA:
ALGEBRAIC CURVES UNIFORMIZED BY CONGRUENCE SUBGROUPS OF TRIANGLE GROUPS

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This note gives some errata and addenda for the article Algebraic curves uniformized by congruence subgroups of triangle groups [1]. Thanks to Juanita Duque Rosero and Michael Schein.

(1) Before Theorem C: replace “n | 6abc” with “n coprime to 6abc”.
(2) Theorem C, Proposition 9.7: it need not follow that the projection onto many PGL$_2$ factors is surjective; rather, only that the image contains a dense subgroup of $\prod_{p \mid N} \text{PSL}_2(\mathbb{Z}_E, p)$.
(3) Lemma 5.5: should be “$(mz, m(z+1), mz(z+1))$” (replace $k$ by $z$).
(4) (5.21): maps to $\text{SL}_2(\mathbb{Z}_F/\mathfrak{n})/\{\pm 1\}$.
(5) Below equation (5.21): replace “Let $n$ be the prime of $E = F(a,b,c)$ below $N$” with “Let $n = \mathbb{Z}_E \cap \mathfrak{n}$ be the prime of $E$ below $\mathfrak{n}$”.
(6) Remark 5.24: sign errors crept into the second generator. The correct orthogonal elements for $B$ are
$$1, 2\delta_a - \lambda_{2a}, (\lambda_{2a}^2 - 4)\delta_b + (\lambda_{2a}\lambda_{2b} + 2\lambda_{2c})\delta_a - (\lambda_{2a}^2\lambda_{2b} + \lambda_{2a}\lambda_{2c} - 2\lambda_{2b}),$$
not
$$1, 2\delta_a - \lambda_{2a}, (\lambda_{2a}^2 - 4)\delta_b + (\lambda_{2a}\lambda_{2b} + 2\lambda_{2c})\delta_a - (\lambda_{2a}^2\lambda_{2b} - \lambda_{2a}\lambda_{2c} + 2\lambda_{2b}).$$

In the corrected basis, we obtain the presentation:
$$B \simeq \left( \frac{\lambda_{2a}^2 - 4, - (\lambda_{2a}^2 - 4)\beta}{F} \right) \simeq \left( \frac{\lambda_{2a}^2 - 4, \beta}{F} \right)$$
when $a \neq \infty$.
(7) Proof of Theorem 9.1: the unipotent case should be allowed in the proof, when $s = \infty$. Replace the start of the middle paragraph by:

Next, we show that orders of $g_1, g_2, g_3$ are $a^3, b^7, c^3$. Let $s = a, b, c$ and write $g$ for the corresponding element. We have $\text{tr} \phi(\delta_s) \equiv \pm \lambda_{2s} (\text{mod } \mathfrak{p})$. If $g = 1$, then the image is commutative, and this possibility was just ruled out. If $s = \infty$, then since $g \neq 1$ and $\lambda_{\infty} = 2$ we must have $g$ unipotent, so $g$ has order $p = s^2$.
(8) Lemma 9.8: since $\text{PSL}_2(\mathbb{F}_4) \simeq \text{PSL}_2(\mathbb{F}_5)$, in fact this lemma holds whenever $\#(\mathbb{Z}_F/\mathfrak{p}) \geq 4$.

REFERENCES


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